Variational analysis of high-frequency radar surface currents using DIVA

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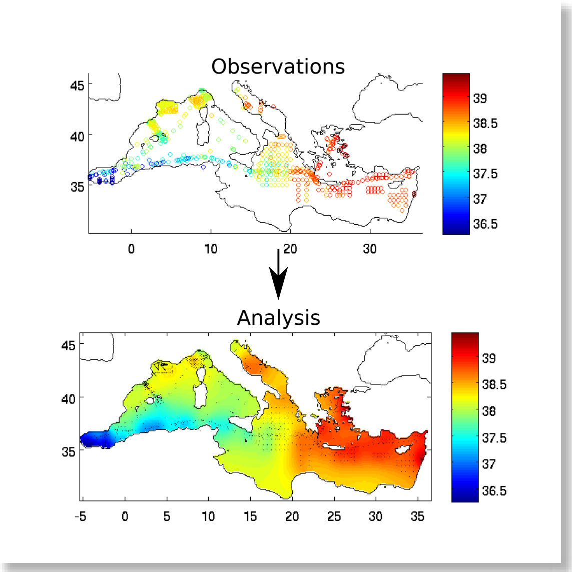
What is DIVA?

- DIVA: Data Interpolating Variational Analysis
- Objective: **derive a gridded climatology from in situ observations**

- The variational inverse methods aim to derive a continuous field which is:
    - **close to the observations** (it should not necessarily pass through all observations because observations have errors)
    - "**smooth**"



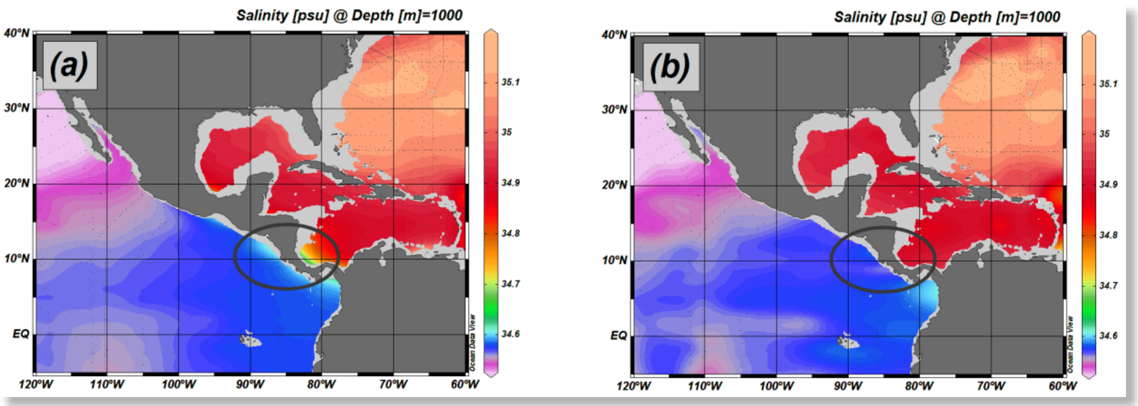
- Formalized via a cost function:

$$J[\varphi] = \sum_{j=1}^{N_d} \mu_j [d_j - \varphi(\mathbf{x}_j)]^2 + ||\varphi - \varphi_b||^2$$

where d_j are the measurements at the location $\mathbf{x}_j$ and their weights μ_j, φ_b is a background estimate of the field.

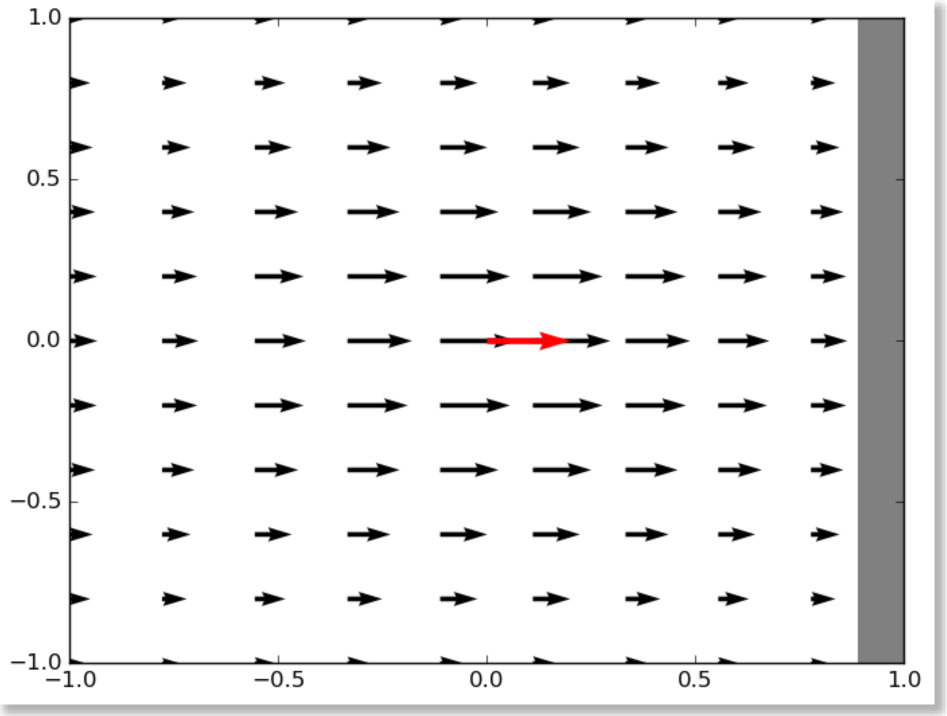
Properties

- decouples basins based on **topography**
- can take **ocean currents** into account
- can detect **trends** in your data
- can detect and remove **outliers**
- consistent **error variance estimation**
- Former version of DIVA: analysis operates in 2 dimensions
- The rewrite DIVAnd does not have this limitation



DIVA and surface currents

- DIVA applied to currents
- Red vector: hypothetical measurement
- Black vectors: analyzed field
- Analysis of radial currents to derive total currents
- Observation operator links the radial currents of the different radar sites



DIVA and surface currents

$$\|\varphi\|^2 = \int_\Omega (\alpha_2 \nabla\nabla\varphi : \nabla\nabla\varphi + \alpha_1 \nabla\varphi \cdot \nabla\varphi + \alpha_0 \varphi^2)\, dx$$

$$J(\mathbf{u}) = \|u\|^2 + \|v\|^2 + \sum_{i=1}^{N} \frac{(\mathbf{u}_i \cdot \mathbf{p}_i - u_{r\,i})^2}{\epsilon_i^2}$$

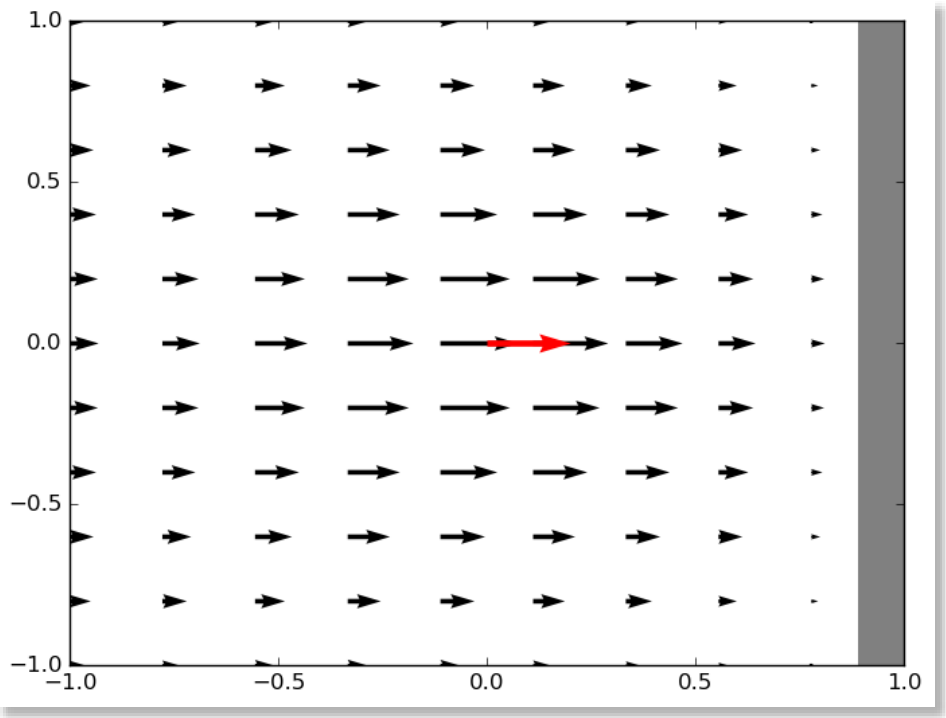
$\mathbf{u} = (u, v)$ and $\mathbf{p}_i$ is the normalized vector pointing toward the correspond HF radar site of the i-th radial observation $u_{r\,i}$

Coastline

- Coastline as a boundary condition
- Constrain at the boundaries $\partial\Omega$

$$\mathbf{u} \cdot \mathbf{n} = 0$$

- Cost functions



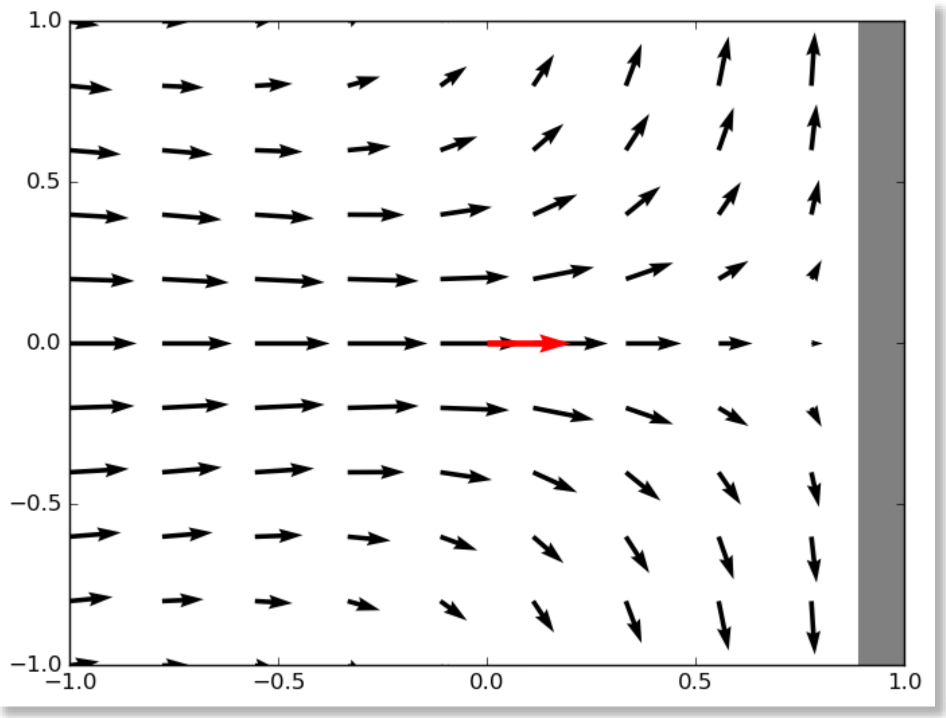
$$J_{\mathrm{bc}}(\mathbf{u}) = \frac{1}{\epsilon_{\mathrm{bc}}^2} \int_{\partial\Omega} (\mathbf{u} \cdot \mathbf{n})^2 ds$$

Horizontal divergence

- Low horizontal divergence of currents

$$\nabla \cdot \mathbf{u} \sim 0$$

- Cost functions



$$J_{\mathrm{div}}(\mathbf{u}) = \frac{1}{\epsilon_{\mathrm{div}}^2} \int_\Omega (\nabla \cdot \mathbf{u})^2 dx$$

Time dimension

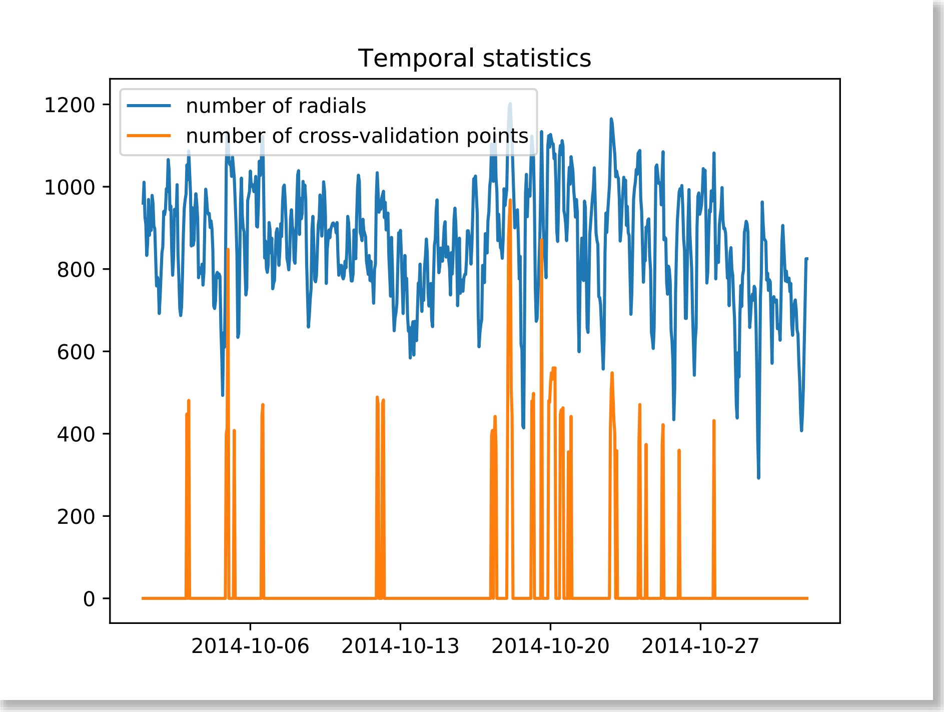
- Either 2D analysis (longitude, latitude) or 3D analysis (longitude, latitude and time)
- 3D analysis:
  - Include the data the hour before and after
  - Temporal correlation length
  - Coriolis force
- Coriolis force and geostrophically balanced mean flow

$$\frac{\partial u}{\partial t} = fv - g\frac{\partial \eta}{\partial x}$$
$$\frac{\partial v}{\partial t} = -fu - g\frac{\partial \eta}{\partial y}$$

- Extend to cost function to include also the surface elevation η

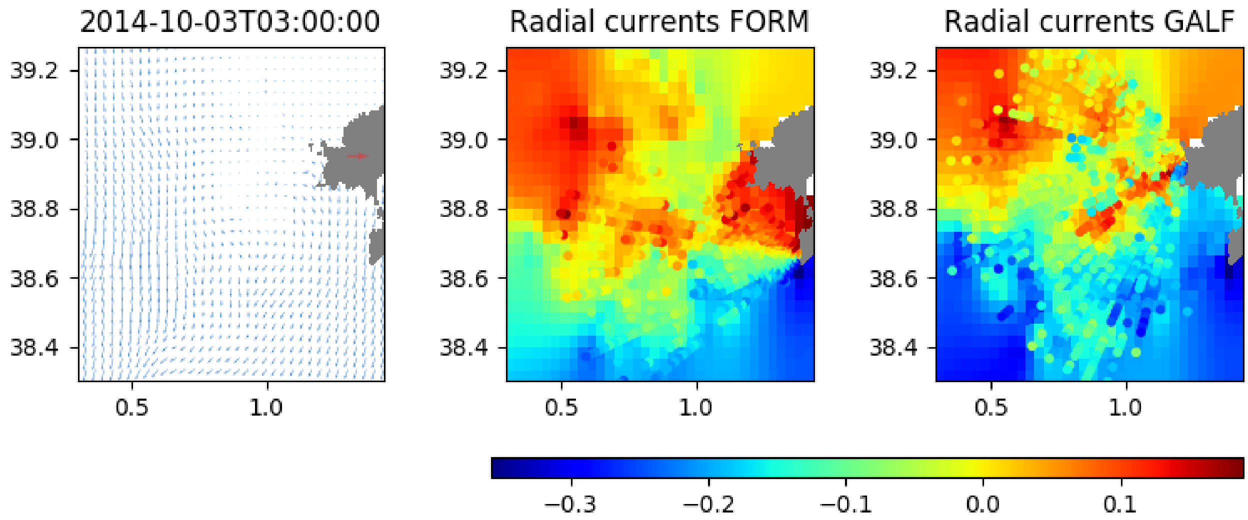
Cross-validation

- HF Radar data from SOCIB (October 2014)
- In 30 current maps with the best coverage we marked some data points as missing (for each radar sites).
- These data points are not used in the following and used to validate the results



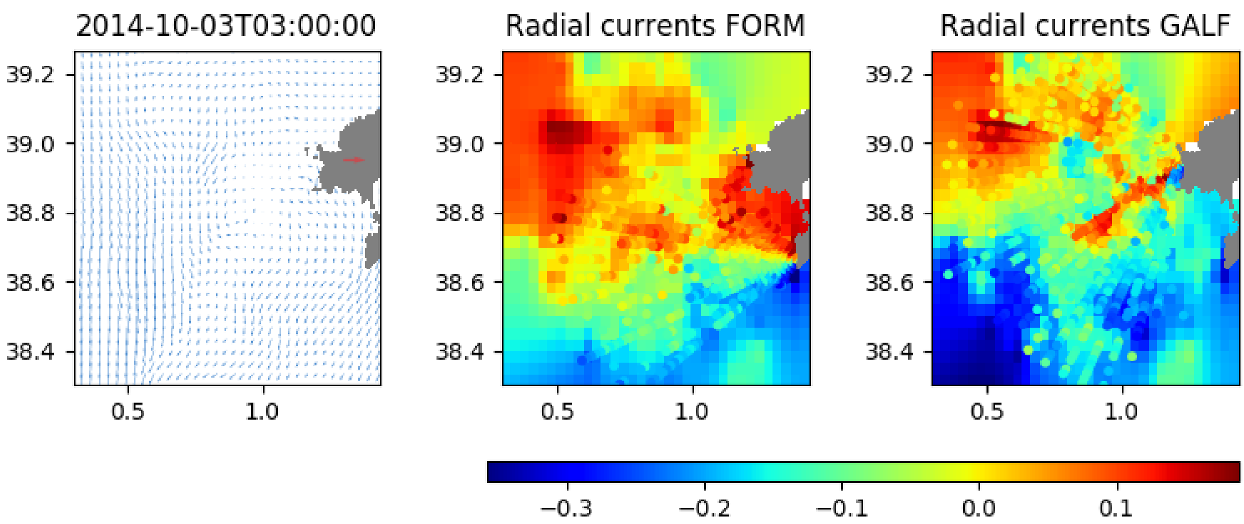
2D Analysis

- Every snapshot is reconstructed individually



3D Analysis

- Including Coriolis force and geostrophically balanced mean flow



Skill score

Skill score S for case C computed as :

$$\mathrm{S}(C) = 1 - \frac{\mathrm{MSE}(C)}{\mathrm{MSE}(2D)}$$

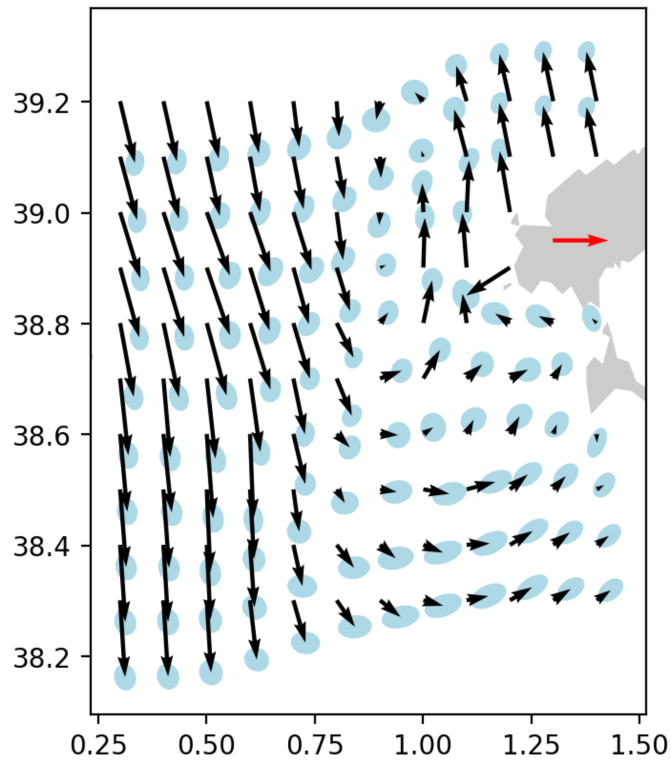
- The 2D case is the base-line for computing the relative improvement
- $\mathrm{MSE}(C)$ is the mean square error (relative to the cross-validation dataset) for the case C.
- If $\mathrm{S} = 0$, the reconstruction is as "good/bad" as the base-line
- If $\mathrm{S} = 1$, the reconstruction is matches perfectly the validation dataset.

Comparison

Case	**Description**	**RMS**	**Skill**
2D	classical 2D-analysis (longitude, latitude)	0.0652	0.000
2D_bc	as 2D, but with boundary conditions	0.0652	-0.000
2D_div	as 2D, but imposing small horizontal divergence	0.0650	0.006
3D	3D-analysis (longitude, latitude, time)	0.0575	0.222
3D_Coriolis	3D-analysis with the Coriolis force	0.0537	0.321
3D_Coriolis_geo	3D-analysis with the Coriolis force and the surface pressure gradient	0.0484	0.450

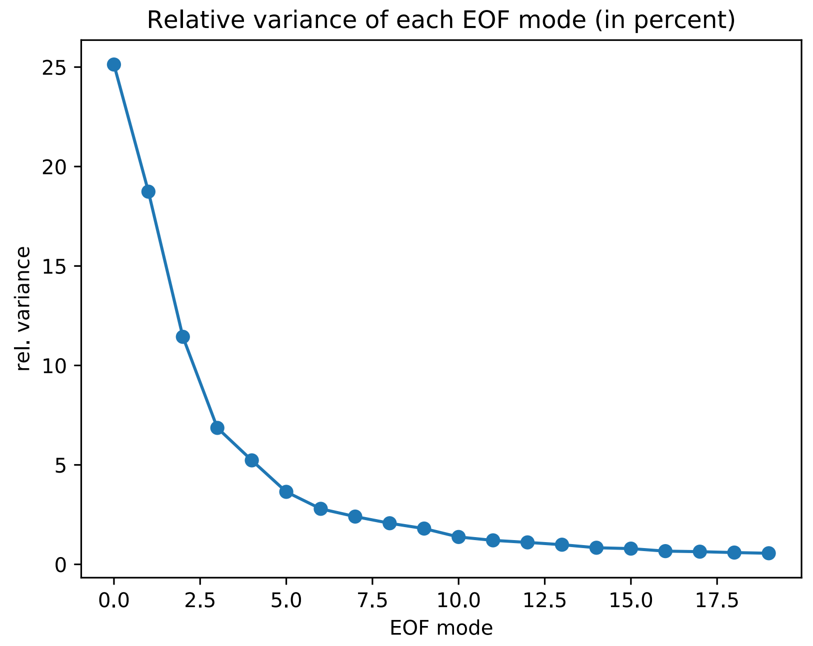
Statistics

- Mean current
- Ellipse representing the standard deviation
- Standard deviation is scaled down by a factor of 5 to enhance visibility
- Variability is quite large compared to the mean current
- The vectors outside the area covered by both antennas are of course much less reliable
- Stung current just in front of Puig des Galfi (GALF)
- Only one current vector is shows for every 3x3 grid cells
- Red arrow represents 0.1 m/s



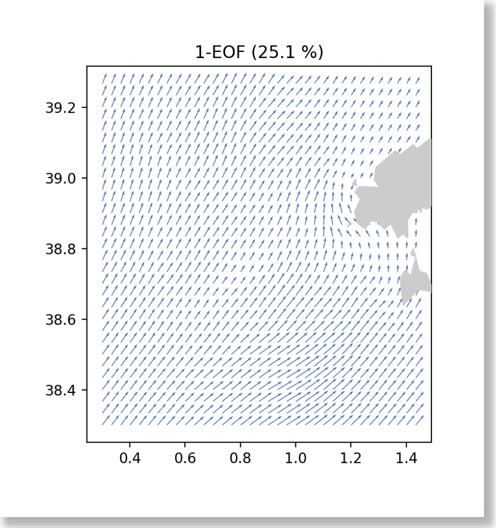
EOF Spectrum

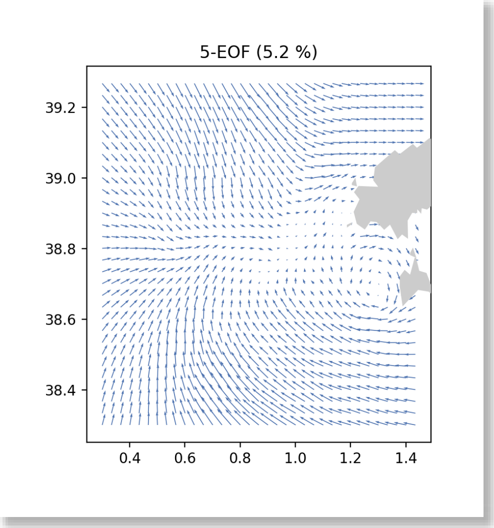
- Variance of the different EOF relative to the total variance
- Velocity EOFs spectra tends to be flatter than spectra derived from e.g. sea surface temperature



EOFs

- In an homogeneous and isotropic environment, EOFs tend to have structures like Fourier modes
- It is rather the departure from these structure which are interesting





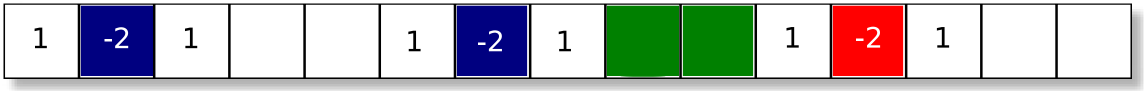
- EOF 1: Relatively uniform motion still affected by the presence of the coastline

- Gyre-like structure in front of Formentera (5-EOF)

Conclusions

- DIVA framework was extended to handle surface currents and able to handle observations when only one component of the velocity vector is measured.
- 2D analyses were used as a base-line for different test cases.
- Including boundary conditions and the constrain on small divergence did not improve the accuracy of the constructions.
- However, taking for every time instance the previous and the following radial maps into account (i.e. a 3D analysis), the skill score could be improved.
- Every time additional dynamical information was added in the analysis the skill score was improved.
- Dynamical information appears to be highly beneficial when analyzing surface currents.

Decoupling in 1D



- First 1D, green: land points, everything else: sea point
- Weights of the discretized Laplacian (in finite difference)

$$\nabla^2 = \frac{\partial^2}{\partial x^2}$$

- Laplacian cannot be computed at the land boundary
- If 3 consecutive (sea) values are equal $\rightarrow$ Laplacian = 0
- Laplacian constrain (and the gradient constrain) forces that every values is close to its right and left neighbor
- This constrain is effective everywhere except near the boundary
- The Laplacian couples directly every grid point with its two neighbors,
- Indirect coupling: two grid points that are separated by some distance as long as they are not separated by land
- The result is that value of the analysis at the two blue points must be close to each other
- However, this is not the case of the blue points and the red point

DIVA updates

- We rewritten DIVA in **Julia** (divand.jl)
  - Curvilinear orthogonal grids
  - Ability to work in n-dimensions
- Julia: good trade-off between **efficiency** of a compiled language and **flexibility** of a dynamic language
- Facilitate the installation:
  - Use **Jupyter notebooks** fully configured environment for divand.jl
  - **Docker container** allows one to easily replicate these environments
- Continue to maintain the Fortran version of DIVA for ODV

Matlab

Python

R

Linear Algebra

Modern programming concepts

Data frames

julia

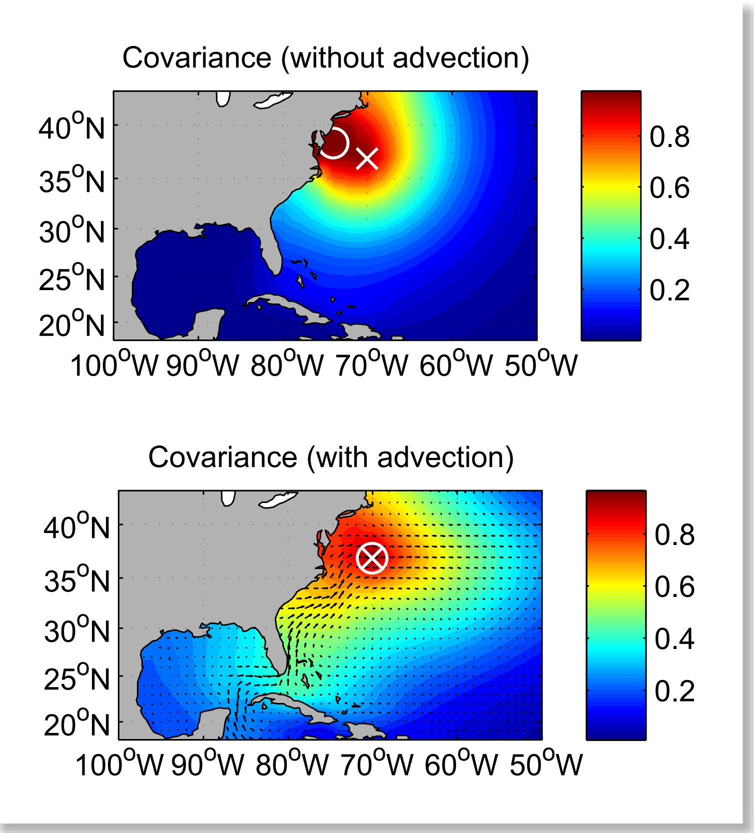
+ Compilation to machine code
+ Performance approaching C
+ Multiple dispatch
+ Type system
+ Lisp-like macros and Metaprogramming

Advection constrain

- The information in the observation can be spread preferentially following currents

$$J_a(\phi) = \int_D (\mathbf{v} \cdot \nabla\phi)^2 dD$$

where $\mathbf{v}$ is a vector field.



Mean

- Strong current just in front of GALF is more coherent when visualizing all vector plots
- Influence of the coast line in steering the flow field