

DIVA

Overview and exercises via web server

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<http://modb.oce.ulg.ac.be/GHER>



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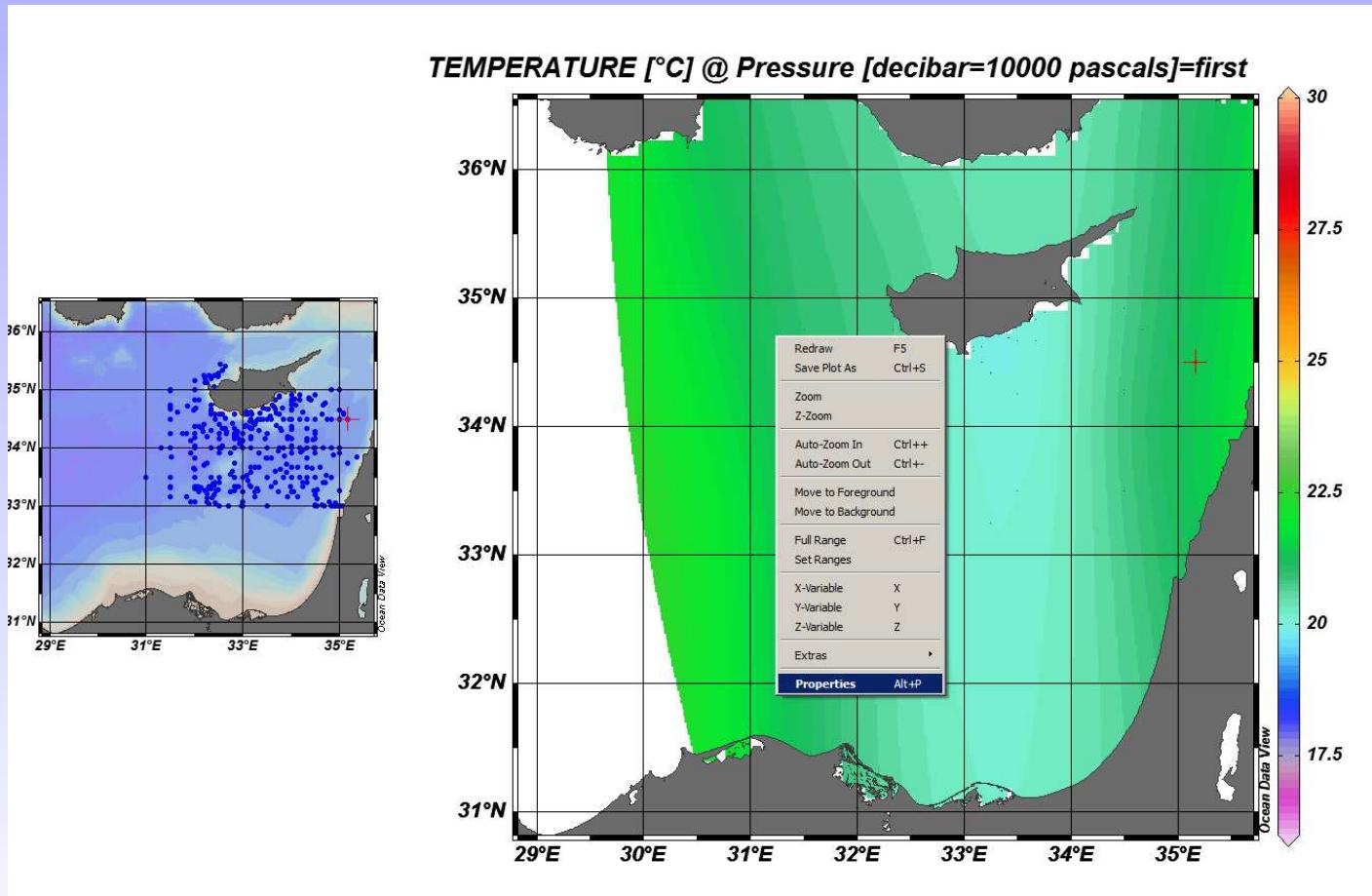
Outline

- ***ODV-DIVA gridding(12:00-12:30)***
- ***Theory (13:30-14:00)***
- ***Implementations (14:00-15:00)***
- ***Diva-on-web (15:30-16:45)***
- ***User feedback (16:45-17:00)***

- ***ODV-DIVA gridding(12:00-12:30)***
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Until lunch

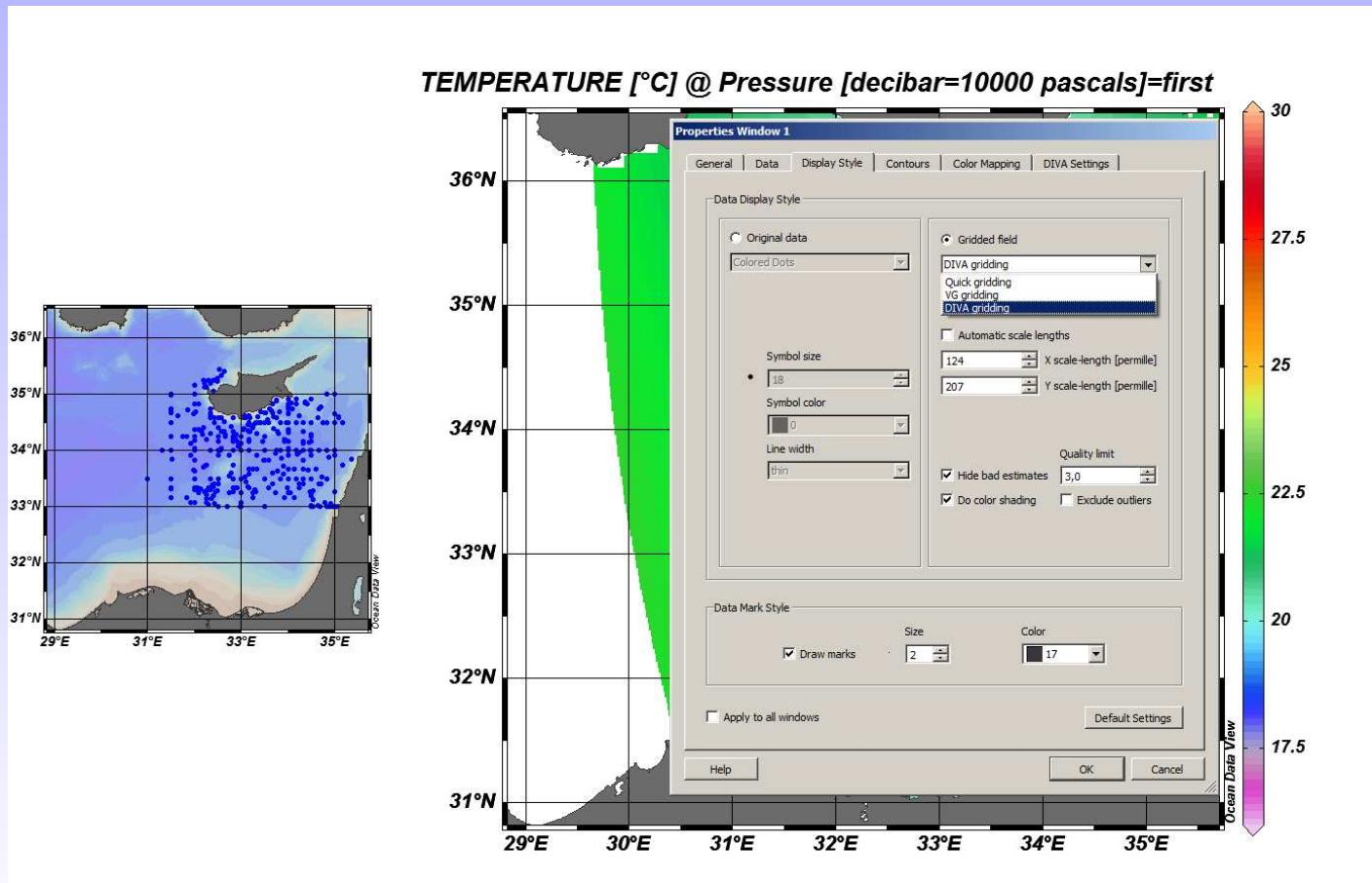
Take the example you worked on when plotting horizontal (or vertical) sections and edit properties (if not available, ask us for the example)



Try parameters of gridding

On Display Style

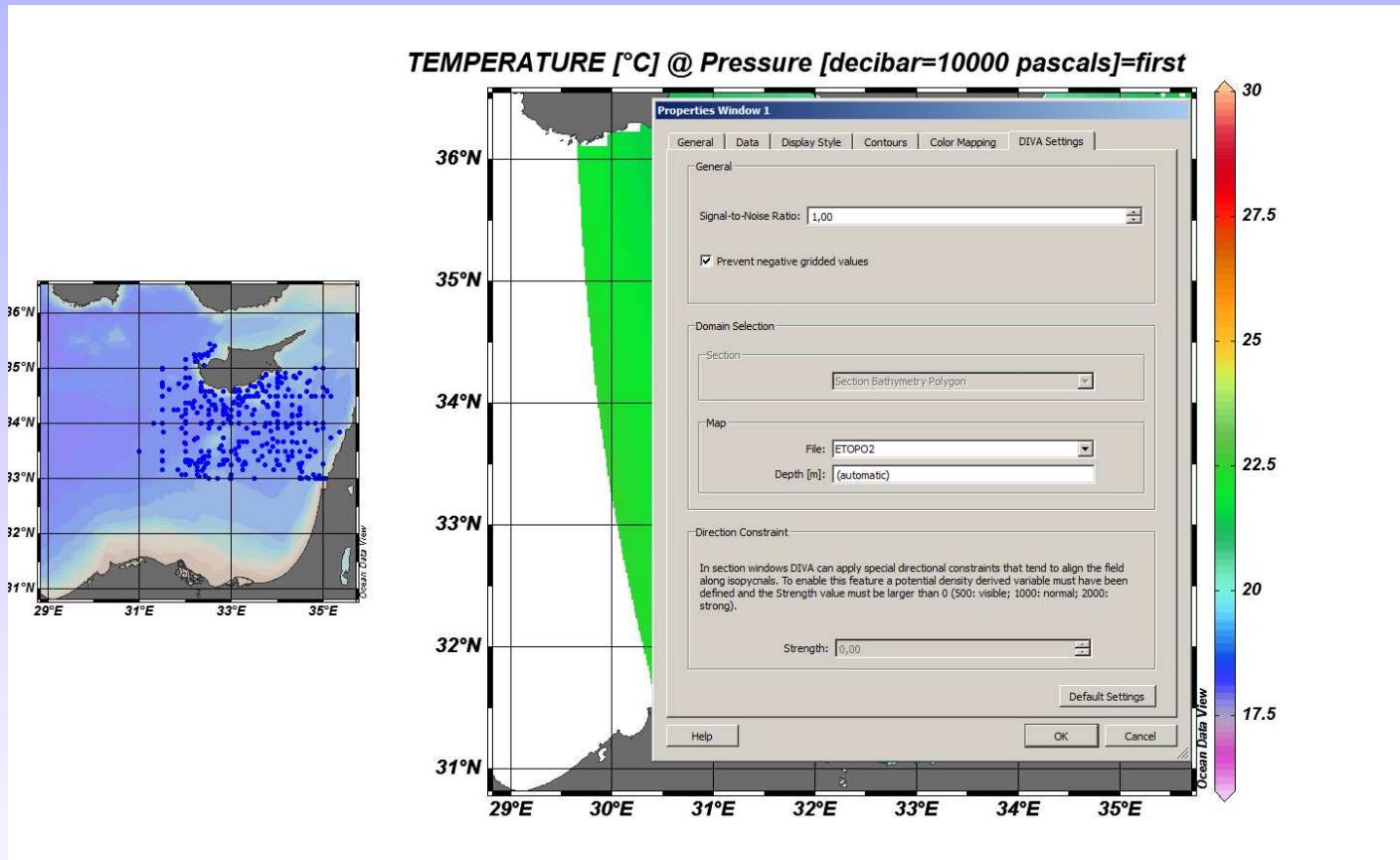
- change analysis method (quick, VG, DIVA)
- change X scale-length and Y scale-length



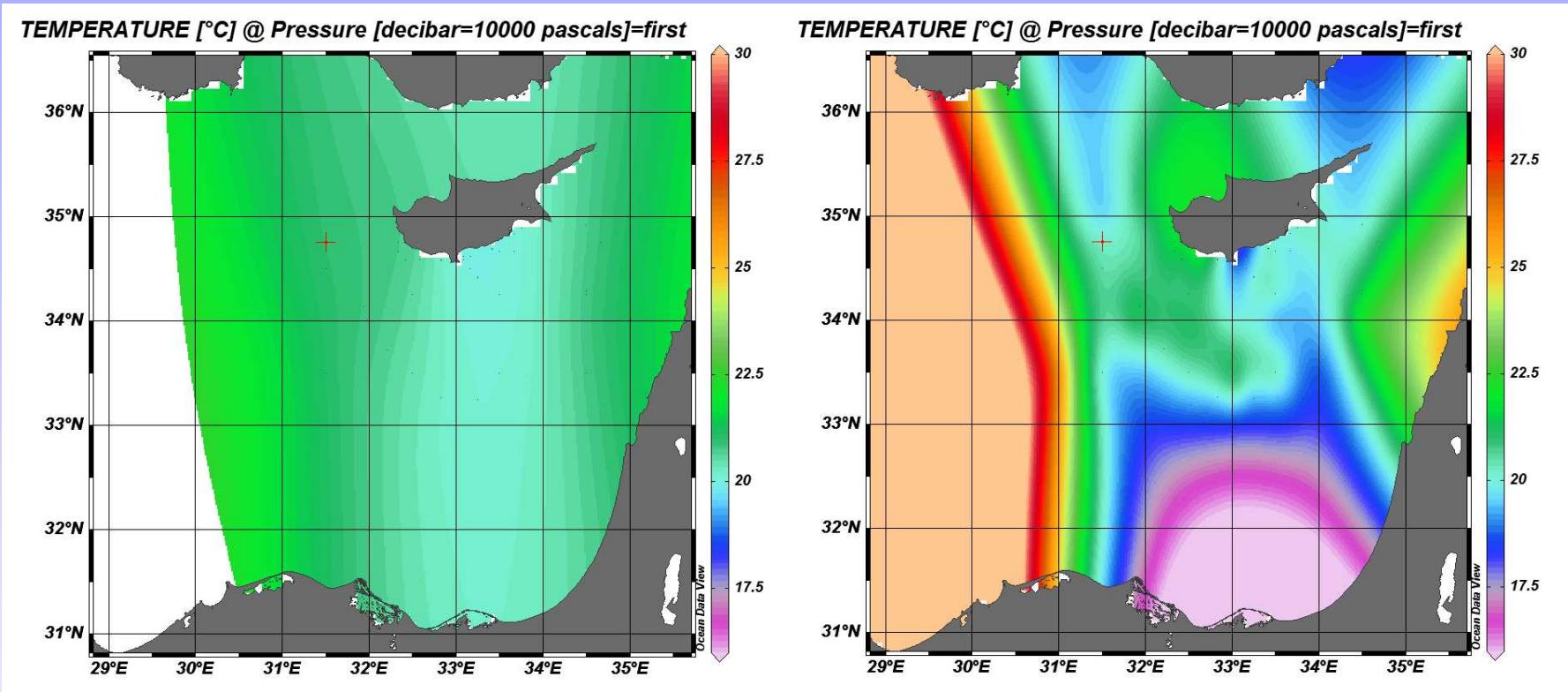
Try parameters of gridding

On DIVA Settings

- Change Signal-to-Noise Ratio
- If available (vertical section), use directional constraint

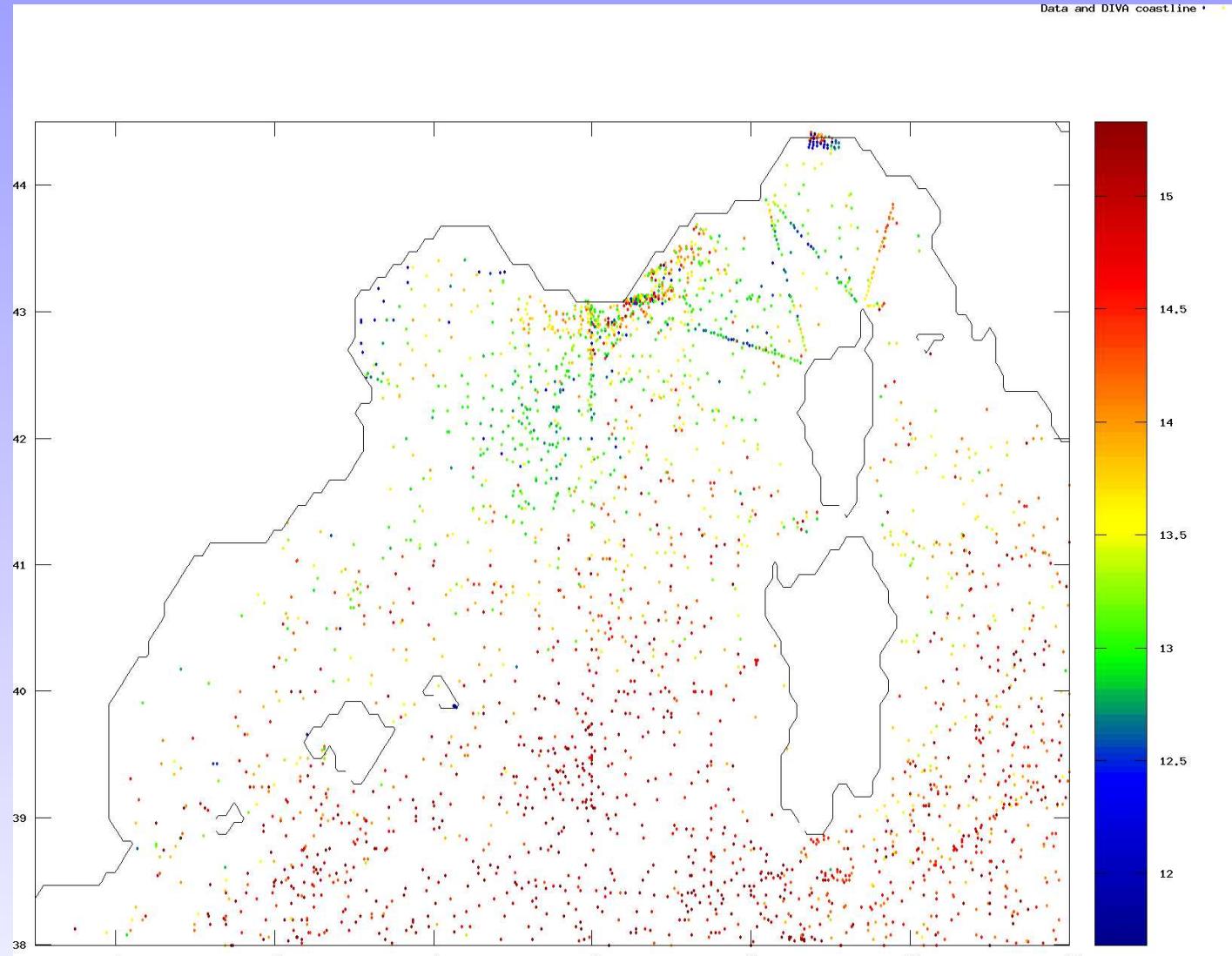


Observed changes in analyses due to changes in parameters (here signal-to-noise ratio from 1 to 100) will be explained after lunch



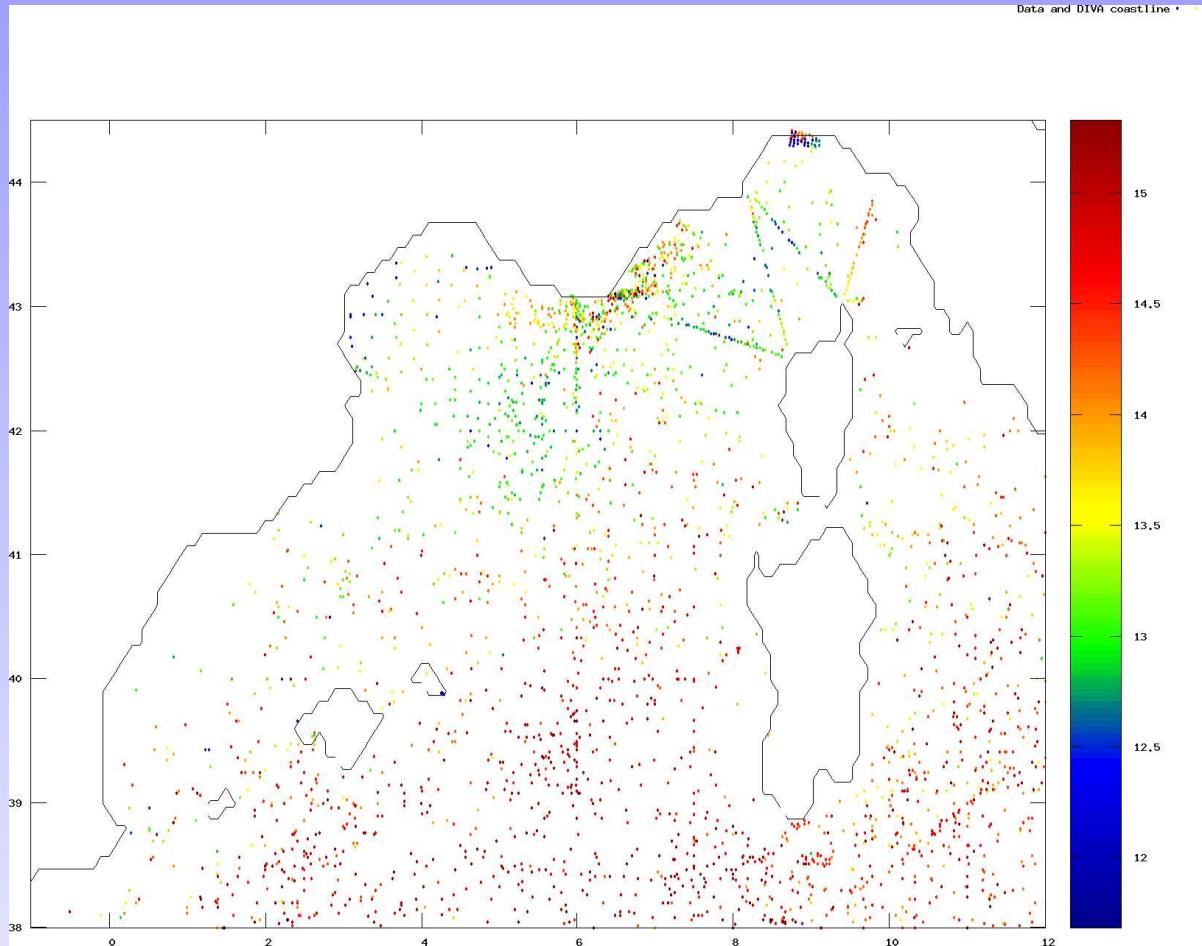
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- *User feedback (16:45-17:00)*

Common problem

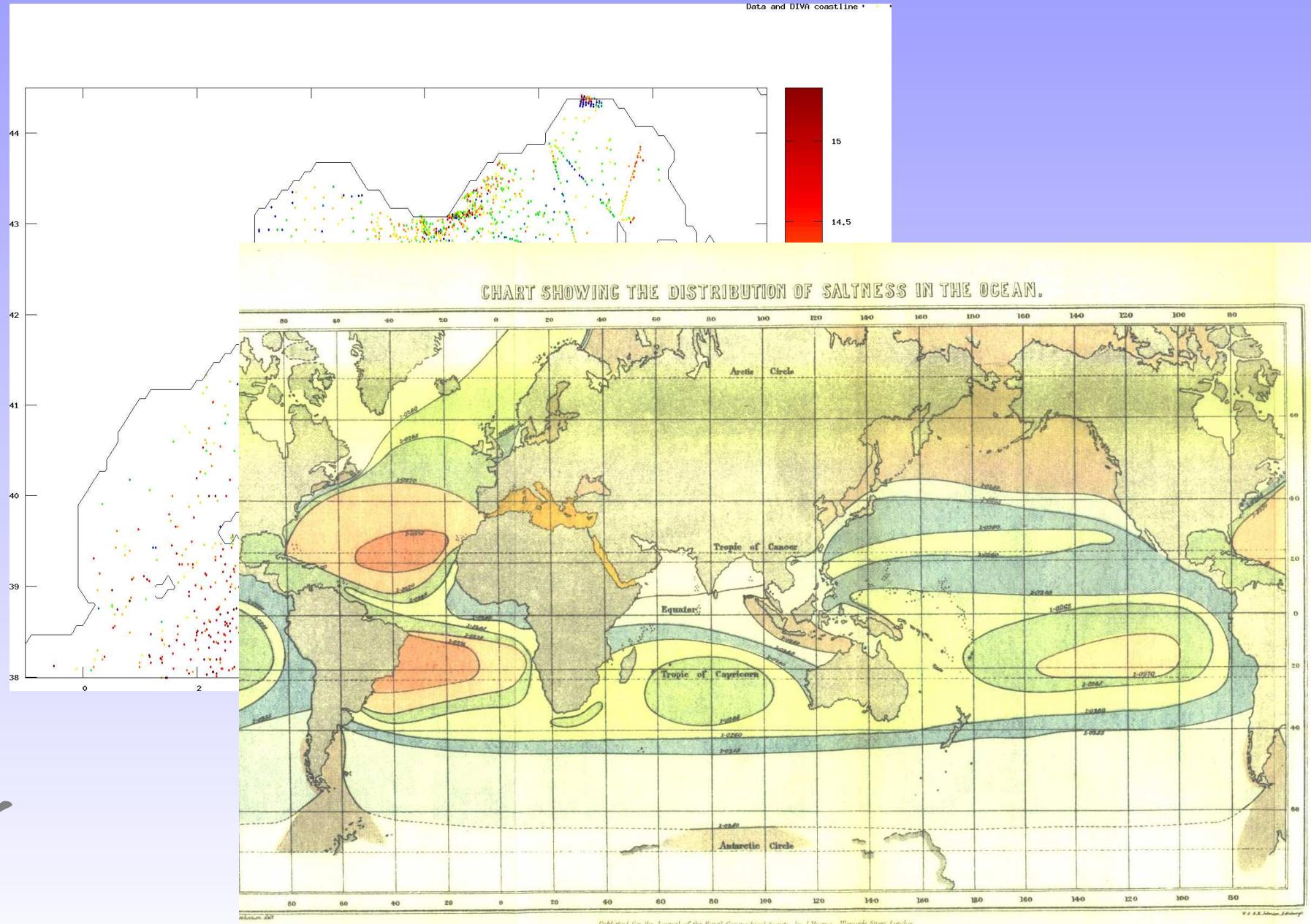


Appears when trying to produce maps, calculate volume averages, prepare initial conditions for models, quality control of data ...

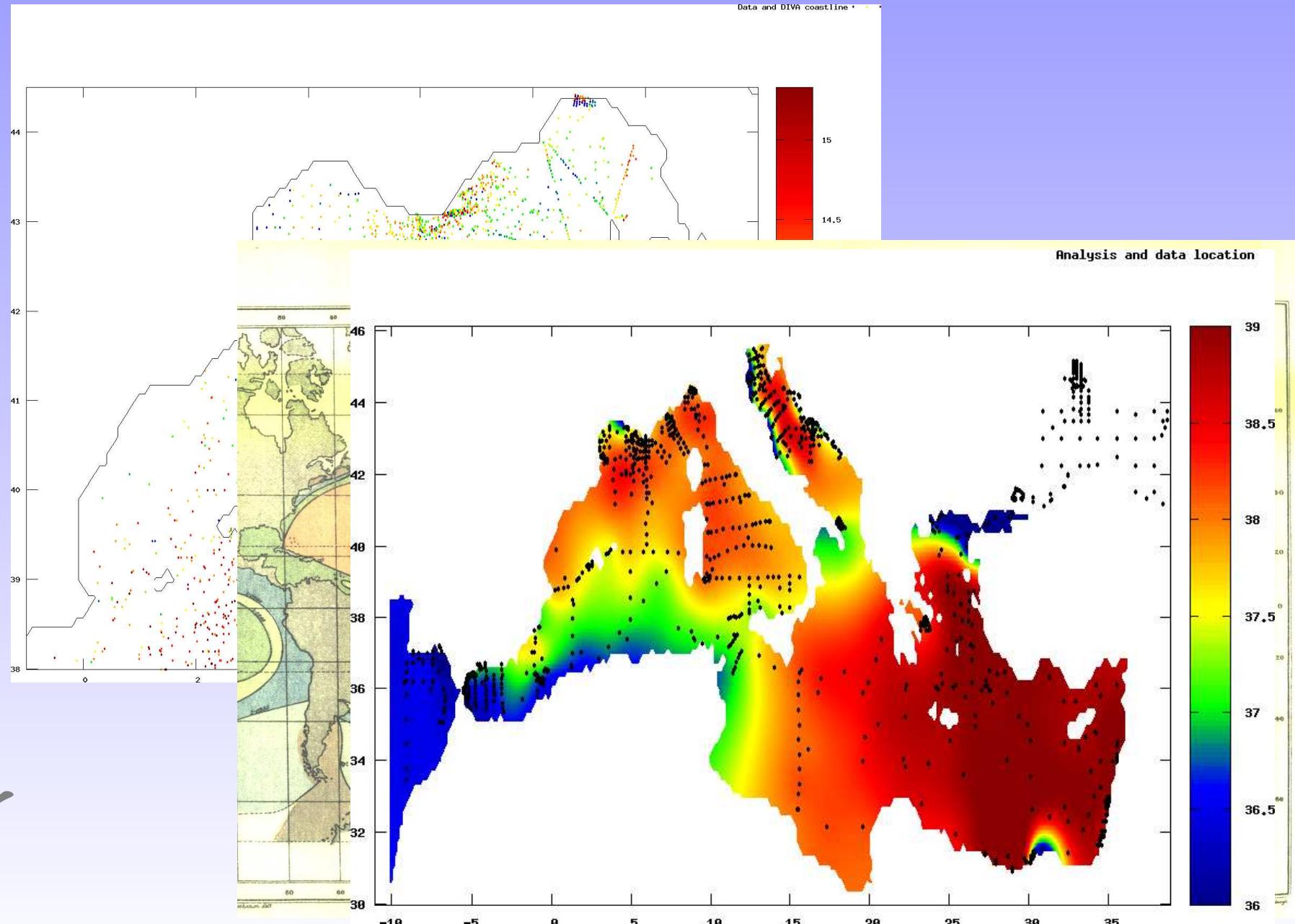
Solutions



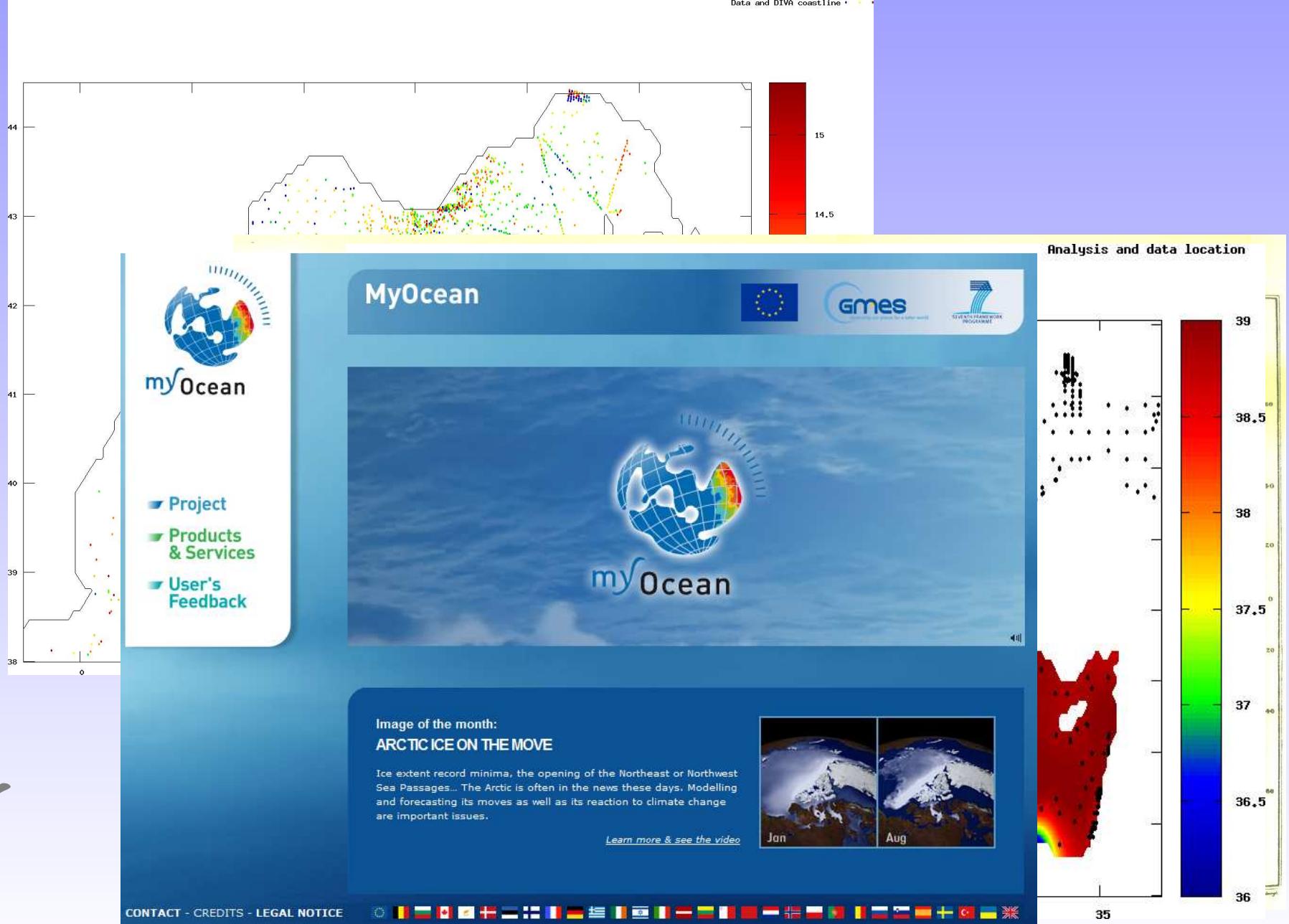
Solutions



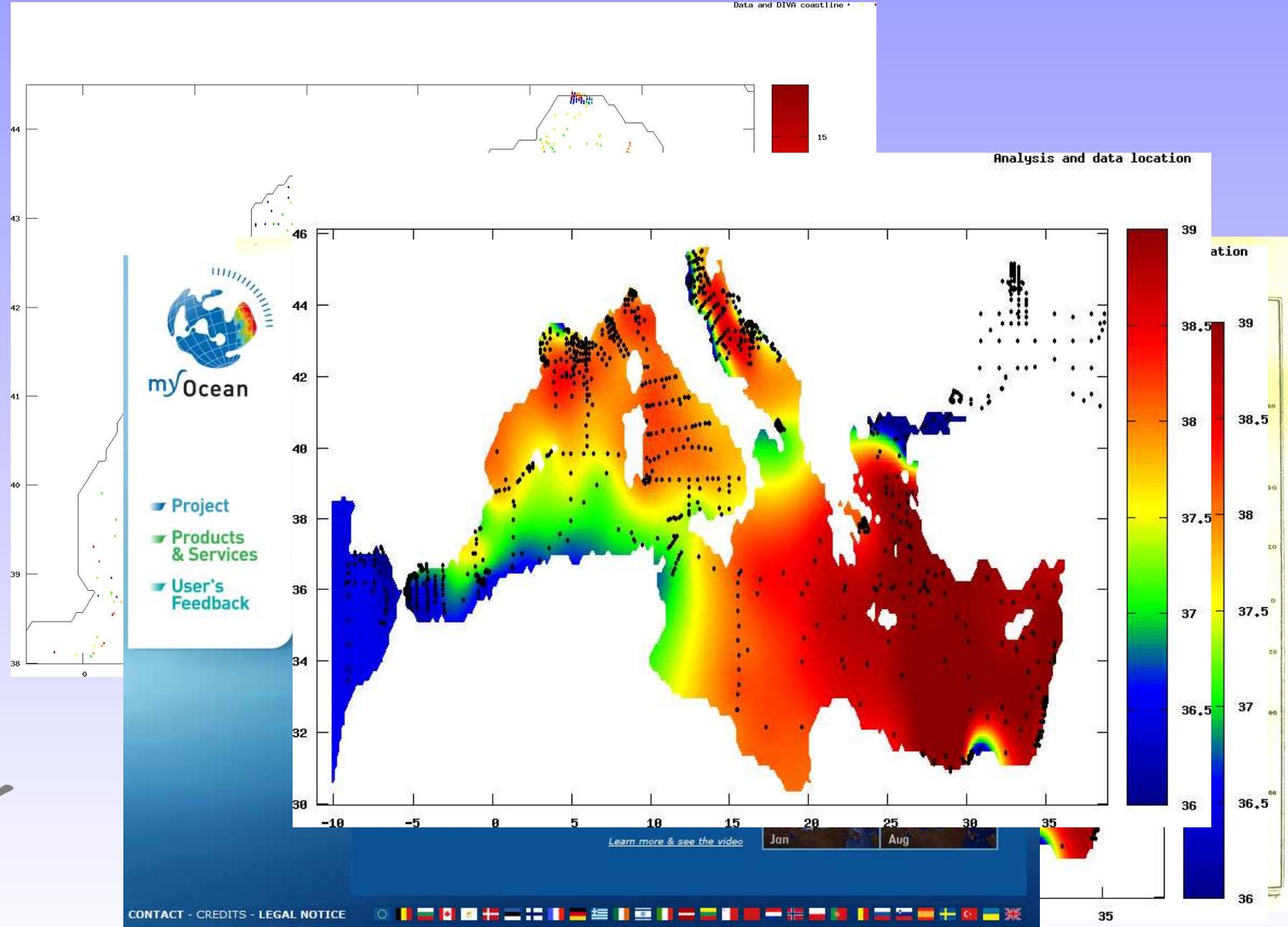
Solutions



Solutions



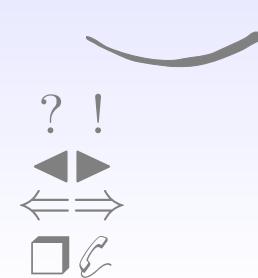
Solutions



Estimation

- Observer 1: 14°
- Observer 2: 16°

Your best guess ?

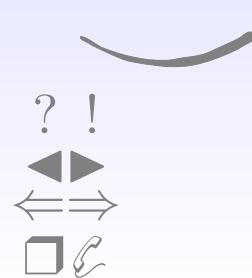


Estimation

- Observer 1: 14°
- Observer 2: 16°

Your best guess ?

15°



Estimation

- Observer 1: 14°
- Observer 2: 16°

Your best guess ?

15°

But what if observer 1 uses digital thermometer and observer 2 his finger ?



Estimation

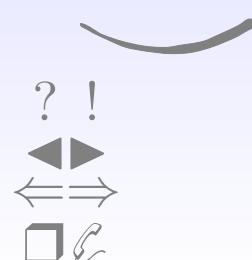
- Observer 1: 14°
- Observer 2: 16°

Your best guess ?

15°

But what if observer 1 uses digital thermometer and observer 2 his finger ?

Best guess probably near 14°.



Estimation

- Observer 1: 14°
- Observer 2: 16°

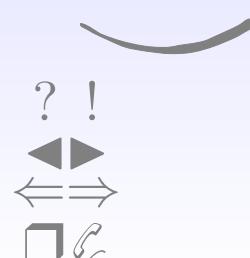
Your best guess ?

15°

But what if observer 1 uses digital thermometer and observer 2 his finger ?

Best guess probably near 14°.

Exploit knowledge of errors !



Optimal estimate

$$T_1 = T^t + \epsilon_1, \quad \langle \epsilon_1 \rangle = 0, \quad T_2 = T^t + \epsilon_2, \quad \langle \epsilon_2 \rangle = 0 \quad (1)$$

statistical average, denoted by $\langle \quad \rangle$ with unbiased estimates $\langle \epsilon_* \rangle = 0$
Linear estimate

$$T = w_1 T_1 + w_2 T_2 = (w_1 + w_2)T^t + (w_1 \epsilon_1 + w_2 \epsilon_2) \quad (2)$$

$$\langle T \rangle = (w_1 + w_2)T^t, \quad (3)$$

we obtain an unbiased estimate of the true state if we take $w_1 + w_2 = 1$. This leaves one parameter free to chose: w_2

Exploit knowledge on errors to find optimal value of w_2

Choice of weighting ?

$$T^a = (1 - w_2)T_1 + w_2 T_2 = T_1 + w_2(T_2 - T_1) \quad (4)$$

while in reality there is an error

$$T^a - T^t = (1 - w_2)\epsilon_1 + w_2\epsilon_2, \quad (5)$$

This error is zero on average but its variance is not zero:

$$\langle (T^a - T^t)^2 \rangle = (1 - w_2)^2 \langle \epsilon_1^2 \rangle + w_2^2 \langle \epsilon_2^2 \rangle + 2(1 - w_2)w_2 \langle \epsilon_1 \epsilon_2 \rangle \quad (6)$$

The actual errors ϵ_1 and ϵ_2 are not known, but the error variance $\langle \epsilon^2 \rangle$ are. Often we can reasonably suppose that the errors ϵ_1 and ϵ_2 are uncorrelated $\langle \epsilon_1 \epsilon_2 \rangle = 0$. The error variance $\langle \epsilon^2 \rangle$ of the analysis is

$$\langle \epsilon^2 \rangle = (1 - w_2)^2 \langle \epsilon_1^2 \rangle + w_2^2 \langle \epsilon_2^2 \rangle. \quad (7)$$

So what ?

Minimisation

$$\langle \epsilon^2 \rangle = (1 - w_2)^2 \langle \epsilon_1^2 \rangle + w_2^2 \langle \epsilon_2^2 \rangle. \quad (8)$$

Naturally, the best estimate for T is the one with the lowest expected error variance and we will use w_2 , which minimizes the right-hand side:

$$w_2 = \frac{\langle \epsilon_1^2 \rangle}{\langle \epsilon_1^2 \rangle + \langle \epsilon_2^2 \rangle} \quad (9)$$

$$T^a = \frac{\langle \epsilon_1^2 \rangle \langle \epsilon_2^2 \rangle}{\langle \epsilon_1^2 \rangle + \langle \epsilon_2^2 \rangle} \left(\frac{T_1}{\langle \epsilon_1^2 \rangle} + \frac{T_2}{\langle \epsilon_2^2 \rangle} \right). \quad (10)$$

Best estimate

With (9) we obtain the minimal error variance

$$\langle \epsilon^2 \rangle = \frac{\langle \epsilon_1^2 \rangle \langle \epsilon_2^2 \rangle}{\langle \epsilon_1^2 \rangle + \langle \epsilon_2^2 \rangle} = \left(1 - \frac{\langle \epsilon_1^2 \rangle}{\langle \epsilon_1^2 \rangle + \langle \epsilon_2^2 \rangle} \right) \langle \epsilon_1^2 \rangle, \quad (11)$$

while the estimate of the temperature itself reads

$$T^a = T_1 + \left(\frac{\langle \epsilon_1^2 \rangle}{\langle \epsilon_1^2 \rangle + \langle \epsilon_2^2 \rangle} \right) (T_2 - T_1). \quad (12)$$

Error variance on the combination of T_1 and T_2 is smaller than both $\langle \epsilon_1^2 \rangle$ and $\langle \epsilon_2^2 \rangle$.

Optimal Interpolation

Same problem but data distributed in space and *a priori* information on background (with variance σ^2).

Weighting of background (zero value when working with anomalies and σ^2 local variance and covariances between points) information and data points (observed values and observational error variance)

- "Model forecast": Background field.
- Need for covariance of the background field between data points: each element i, j of \mathbf{B} provides the covariance between points in location i and j . Covariance between a given point and all data points is stored in column vector \mathbf{c} and the local variance at the analysis point is noted σ^2 .
- Analysis ϕ of anomaly \mathbf{y} with respect to background leads to spatial analysis at any desired location of covariance between any two points is known.

$$\phi = \mathbf{c}(\mathbf{B} + \mathbf{R})^{-1}\mathbf{y} \quad (13)$$

Background covariance

Problem, how to specify background covariances (between all data points and between data points and the desired analysis location).

- c_i = covariance between location of the analysis and data location of point $i = C(x, x_i)$
- B_{ij} = covariance between location of data point i and location of point $j = C(x_i, x_j)$

Approaches

- Normally obtained via statistics on data. Seldom possible (noticable exception: satellite images).
- Standard OI: via functions $B_{ij} = f(r/L)$ where r is the distance between points i and j , but still function f needs to be determined. L is the so-called correlation length. Here statistics on all data couples as a function of distance.
Example: $f = \sigma^2 \exp(-r^2/L^2)$.
- Via functionals (see [Kernel](#) of DIVA later)

Signal to noise ratio

$$\mathbf{B} = \sigma^2 \tilde{\mathbf{B}} \quad (15)$$

$$\mathbf{R} = \epsilon^2 \tilde{\mathbf{R}} \quad (16)$$

$$\mathbf{c} = \sigma^2 \tilde{\mathbf{c}} \quad (17)$$

with non-dimensional correlation matrixes $\tilde{\mathbf{B}}$, $\tilde{\mathbf{R}}$, $\tilde{\mathbf{c}}$

$$\phi = \tilde{\mathbf{c}} \left(\tilde{\mathbf{B}} + \frac{1}{\lambda} \tilde{\mathbf{R}} \right)^{-1} \mathbf{y} \quad (18)$$

with signal-to noise ratio

$$\lambda = \frac{\sigma^2}{\epsilon^2} \quad (19)$$

Also the error field is only depending on the ratio.

DIVA Basics

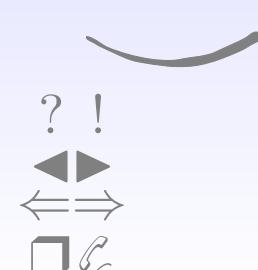
Variational Inverse Method, (Brasseur *et al.*, 1996). Knowing data d_j at location (x_j, y_j) , search the field φ which minimizes

$$J[\varphi] = \sum_{j=1}^{Nd} \mu_j [d_j - \varphi(x_j, y_j)]^2 + \|\varphi - \varphi_b\|^2 \quad (20)$$

$$\|\varphi\| = \int_D (\alpha_2 \nabla \nabla \varphi : \nabla \nabla \varphi + \alpha_1 \nabla \varphi \cdot \nabla \varphi + \alpha_0 \varphi^2) dD \quad (21)$$

The background field φ_b is typically the data average value.

- α_0 penalizes the field itself (anomalies),
- α_1 penalizes gradients $\nabla \varphi$ (no spatial trends),
- α_2 penalizes variability (regularization of second derivatives $\nabla \nabla \varphi$),
- α_* can be related to a length scale L of the analysis,
- μ_j penalizes data-analysis misfits (objective).



Basics

$$\mu = \frac{\sigma^2}{\epsilon^2} \frac{4\pi}{L^2} \quad (22)$$

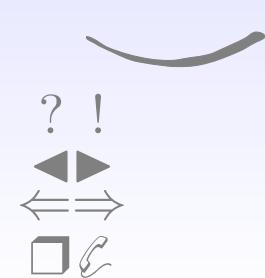
where the σ^2/ϵ^2 is known as a signal to noise ratio S/N .



Solution by finite element method. Note decoupling of subbasins.
(Each element is in fact composed by three sub-elements, each one with cubic functions)

Bad news ☹

- No error estimate comes with the method, only an indicator of data-coverage.



Bad news 😞

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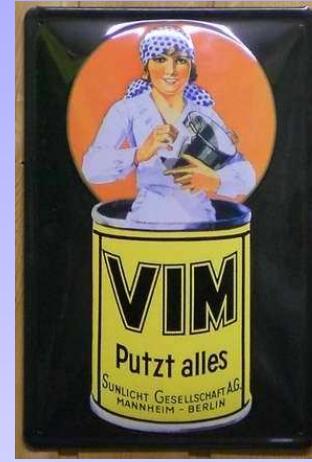


- VIM brand already taken:



Bad news ☹

- No error estimate comes with the method, only an indicator of data-coverage.



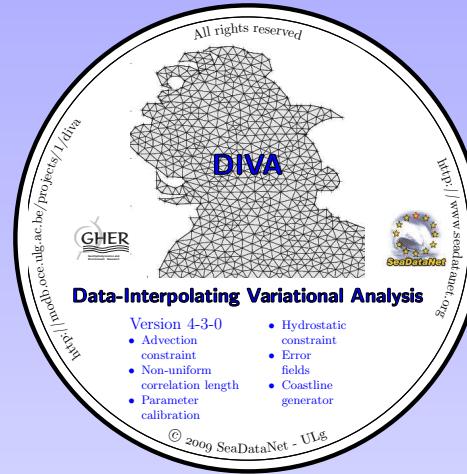
- VIM brand already taken:



- Method is equivalent to well established existing optimal interpolation (OI)

Good news ☺

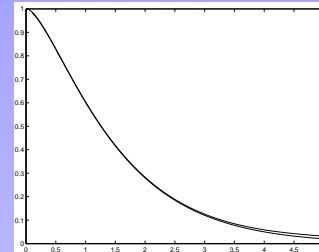
- Name easily changed: DIVA (Data-Interpolating Variational Analysis)



- Optimal interpolation (OI) provides error estimates, so DIVA can also provide it via equivalence
- DIVA has some practical advantages over OI

DIVA as OI

DIVA is identical to the well known Optimal Interpolation



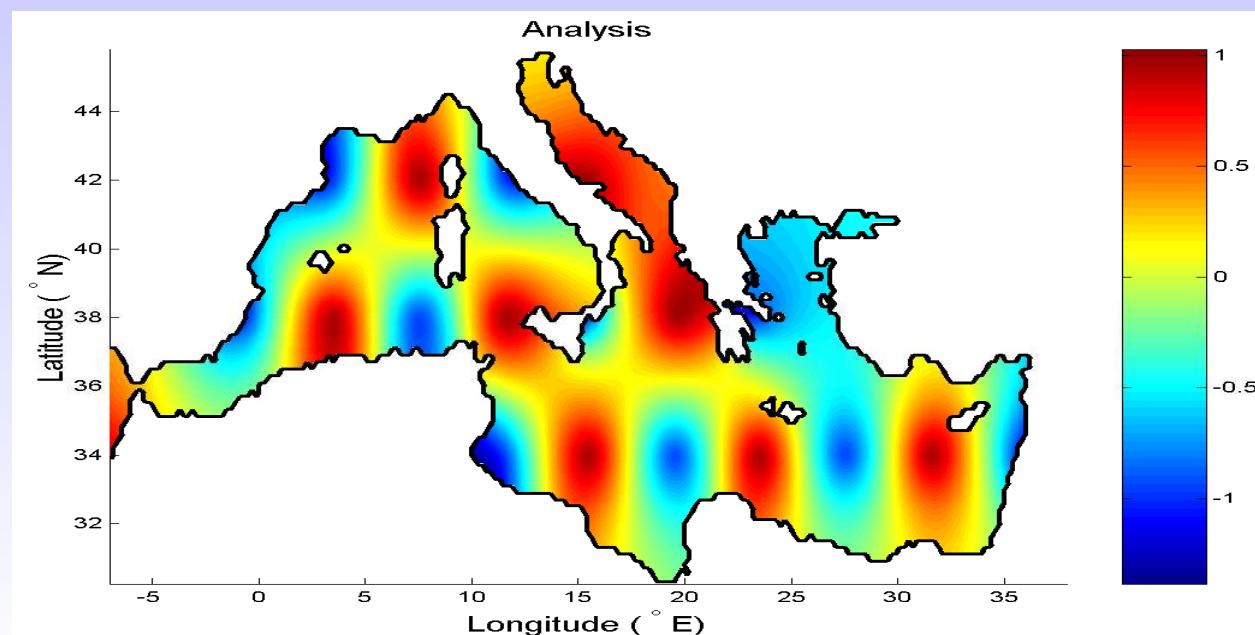
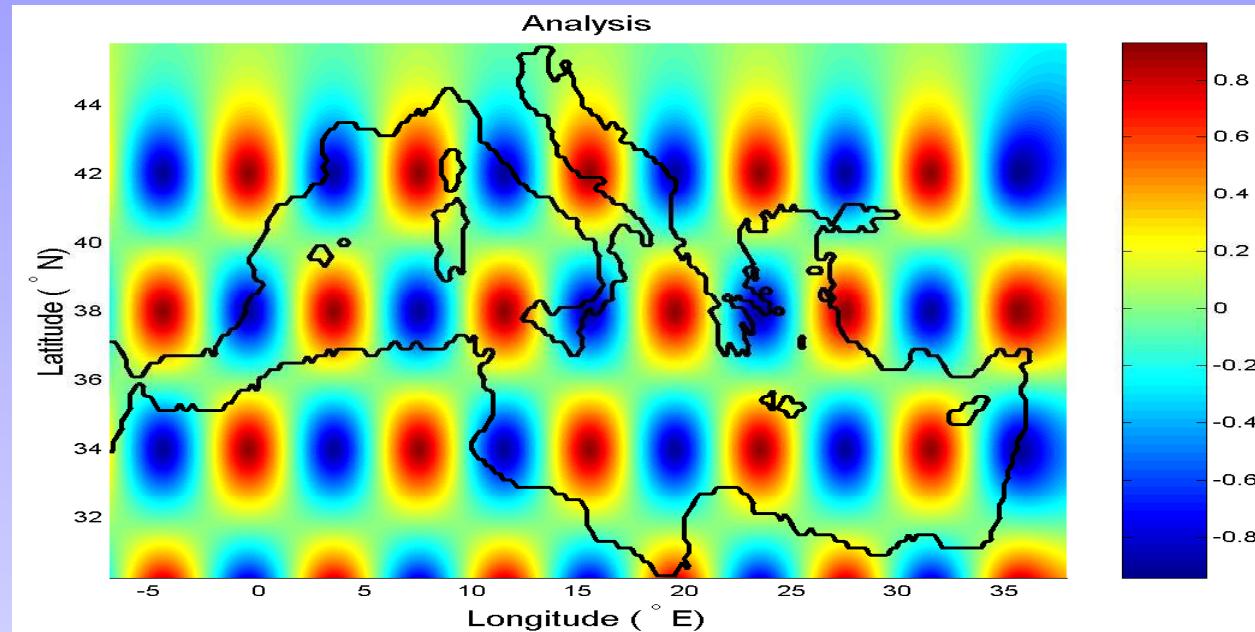
- if so-called reproducing kernel of the norm = covariance function of OI,
- if the noise is random, spatially uncorrelated and the signal/noise ratio parameter is identical with OI.

In this case, the OI solution = DIVA solution.

- Advantages of DIVA: regularization, fast finite-element solution, boundary effects taken into account.
- Difficulties: generalizations to 3D and multivariate versions are "hybrid".

Major direct advantage of DIVA: matrix to invert is related to the finite-element mesh, NOT the number of data. Useful for large data sets (Rixen *et al.*, 2000). Equivalence allows to calculate error fields with DIVA even if formulation does not rely on error minimisation.

Illustration of covariance functions



Comparison

Method	$\min(\epsilon^2)$	3D	Multivar	Ops/image	$\epsilon(r)$	a priori	C.V.	anisotropy
Cressman		★	★	$N_d N_a$		$w(r/L)$	(L)	(★)
O.I.	★	★	★	$N_d^3 + N_d N_a$	★	$c(r/L)$	$L, \sigma^2/\mu^2$	(★)
DIVA	★	(★)	(★)	$N_e^{5/2}$	★	$K(r/L)$	$L, \sigma^2/\mu^2$	★
DINEOF	(★)	★	★	$N_a^{5/4}$	(★)	stat.	N	★

N_d : number of data points

N_a : number of grid points for analysis

N_e : number of finite elements

N : number of EOFs

L : correlation length

σ^2/ϵ^2 : signal to noise ratio

★ : available feature

(★) : available with some adaptations

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Software download

<http://modb.oce.ulg.ac.be/mediawiki/index.php/DIVA>

The research leading to the development of DIVA has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement n°283607, SeaDataNet 2.

Contents [hide]

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- 2 How to get the code?
- 3 User workshops
- 4 The method
- 5 Publications & documents
- 6 Matlab/Octave tools
- 7 Quality control of products
- 8 Testing new versions
- 9 SVN commands for Diva

In a few words

Diva is a software developed for gridding in situ data. It uses a finite-element method to solve a variational principle which takes into account

1. the distance between analysis and data (*observation constraint*),
2. the regularity of the analysis (*smoothness constraint*),

Additions to basic tool

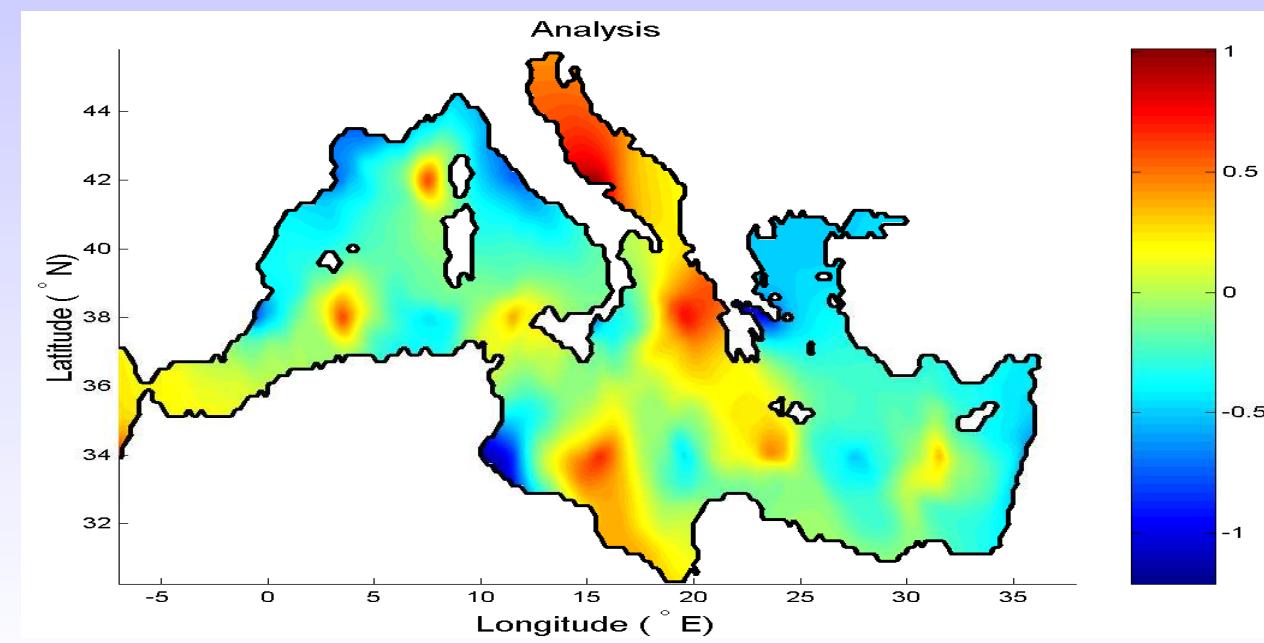
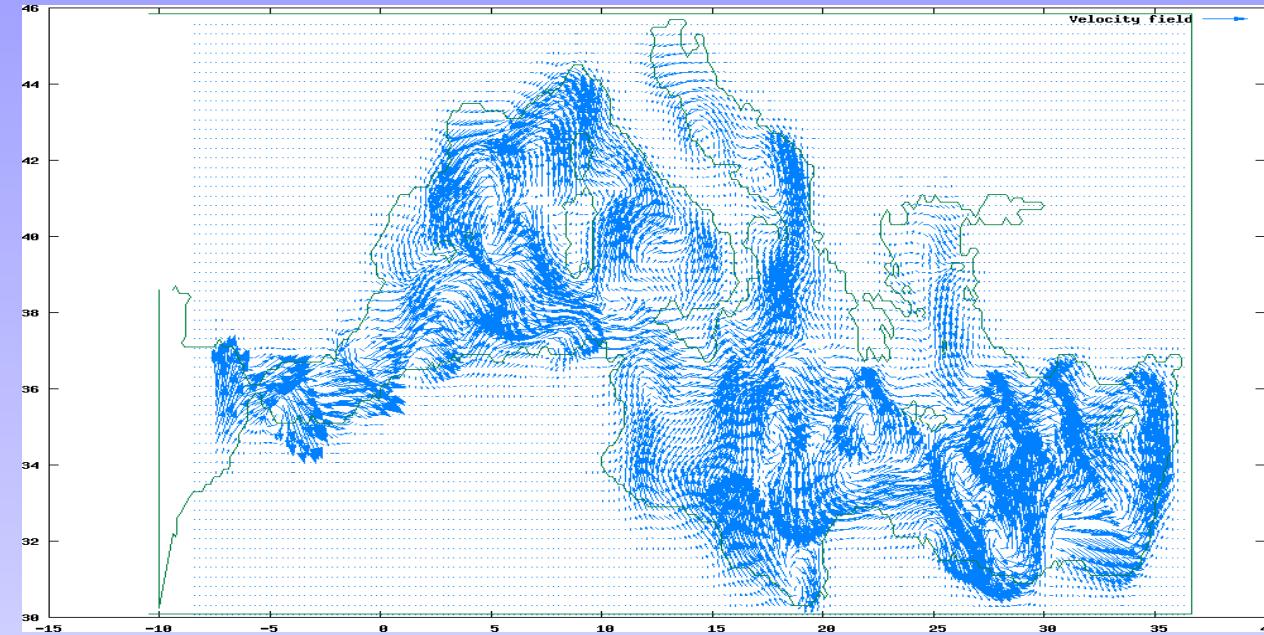
Advection constraint: Augmented cost function to deal with preferred correlation directions, eg, via advection with velocity \mathbf{u} and diffusion \mathcal{A}

$$J_a = J(\varphi) + \theta \int_D [\mathbf{u} \cdot \nabla \varphi - \mathcal{A} \nabla \cdot \nabla \varphi]^2 dD \quad (23)$$

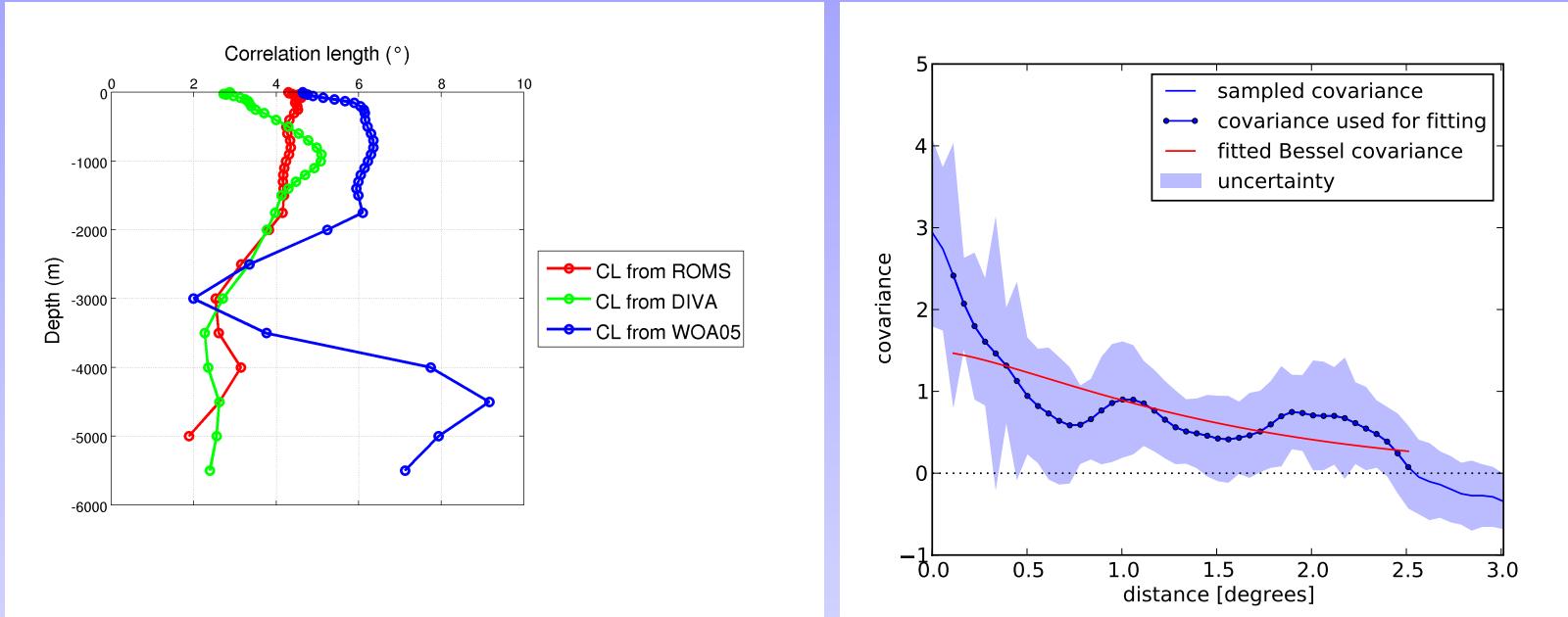
Other features

- Error fields taking data distribution into account.
- Toolbox approach allowing to design own versions.
- 3D and 4D modes by looping, hydrostatic constraint in 3D mode.
- Cross validation tools to infer statistical parameters and error estimates.
- Climatology production version with heterogeneous data distributions [\(detrending\)](#).
- Outlier detection.
- ...

Covariances with advection



Parameter calibration

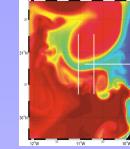


Spatial coherence of parameters: here correlation length obtained with covariance fitting (Troupin *et al.*, 2010).

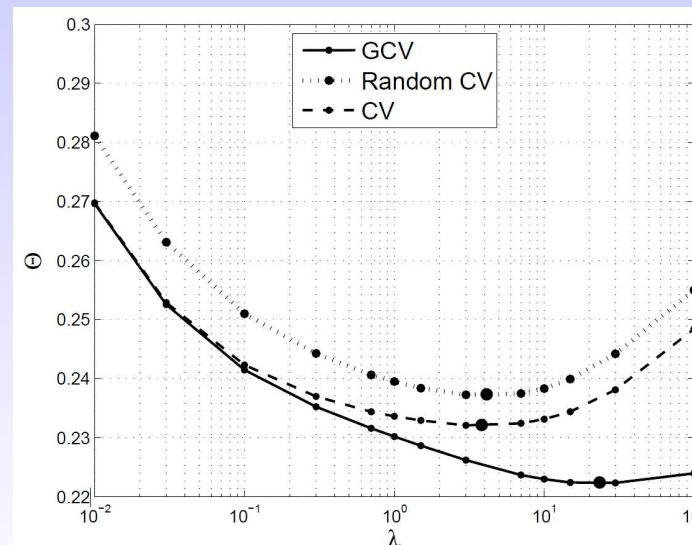
Signal to noise ratio

The most elusive parameter.

- Noise is not only instrumental error:
- Very hard problem to decide on value with dependent data (cross-validation approaches fail).

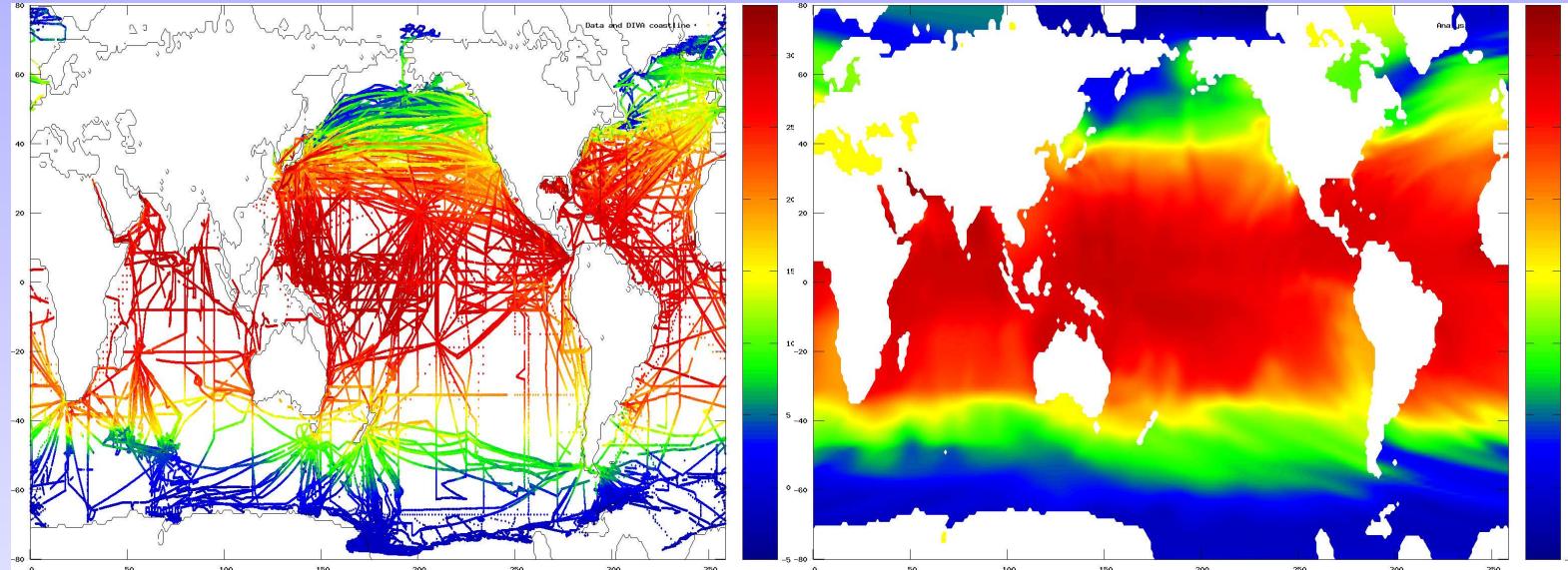


A series of estimation tools are provided with DIVA, but here the experience of oceanographers is critical. A posteriori analysis of residuals allows to verify coherence. With reasonable amount of data, parameter not critical for analysis but for error estimates.

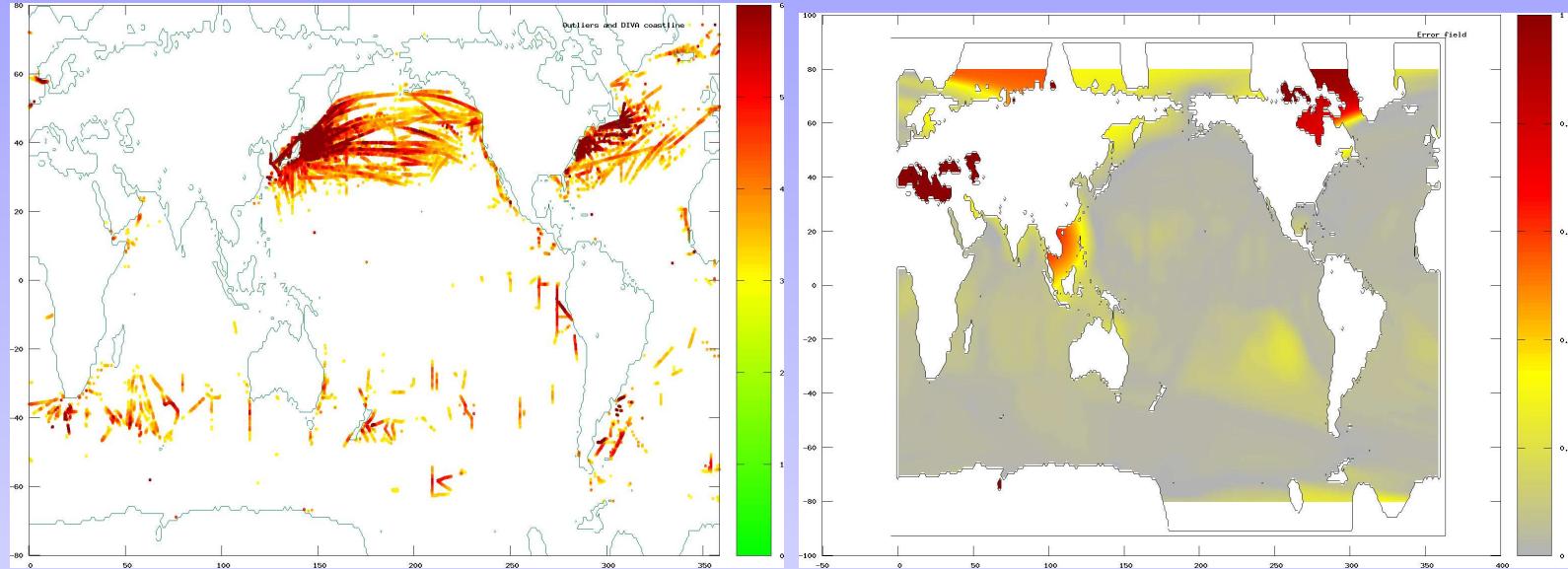


Huge problems

LDEO data base with $4.5 \cdot 10^6$ measurements (Takahashi *et al.*, 2009). Running on a laptop within a few minutes. Shown here, temperature fields.



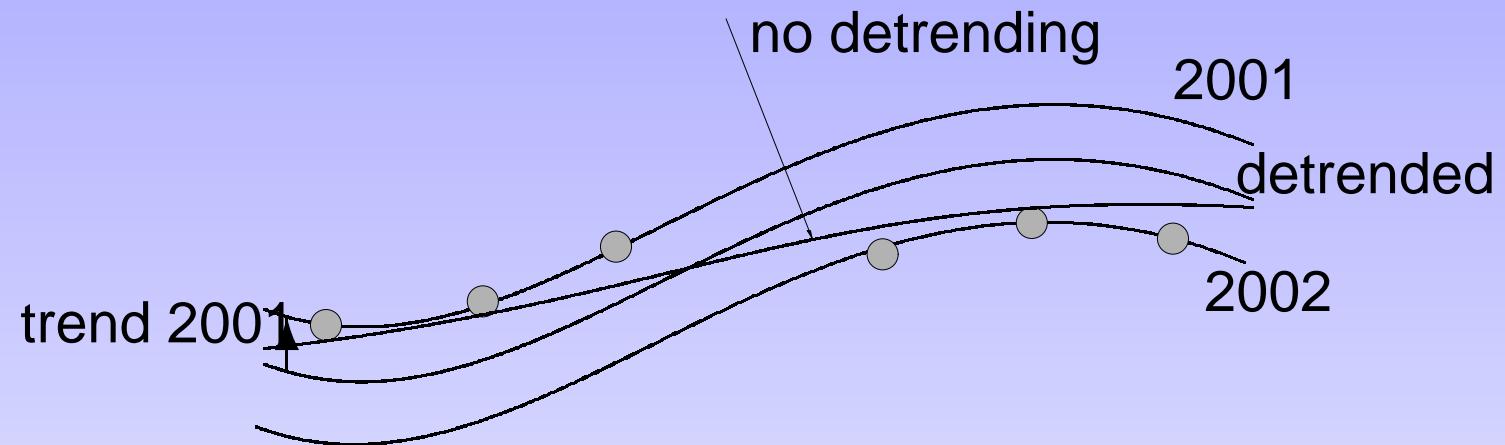
Huge problems, outliers and relative error field



Outliers detected via comparison of statistically expected residuals (value provided by the DIVA analysis) and actual residuals.

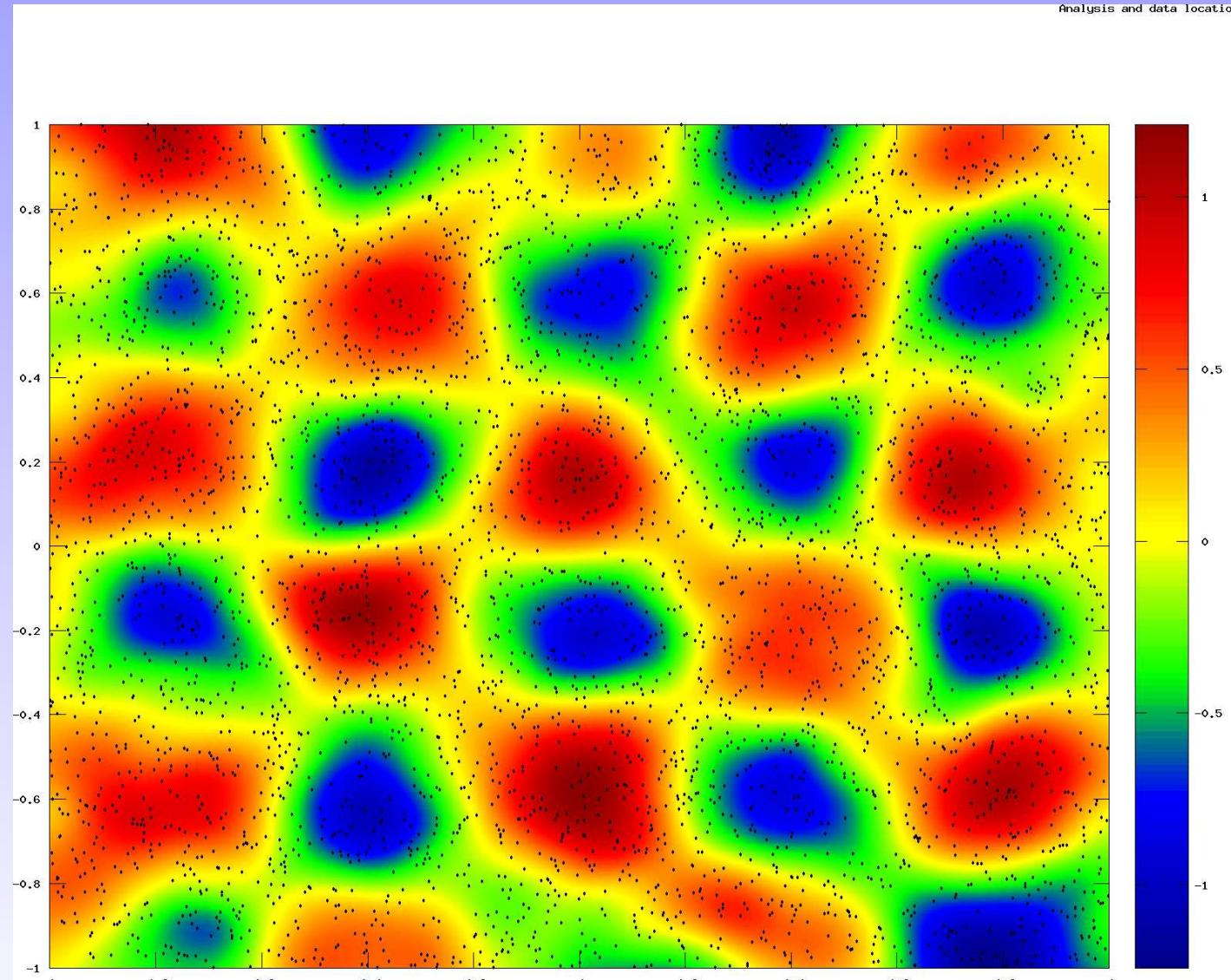
Detrending

Heterogeneous data distribution:



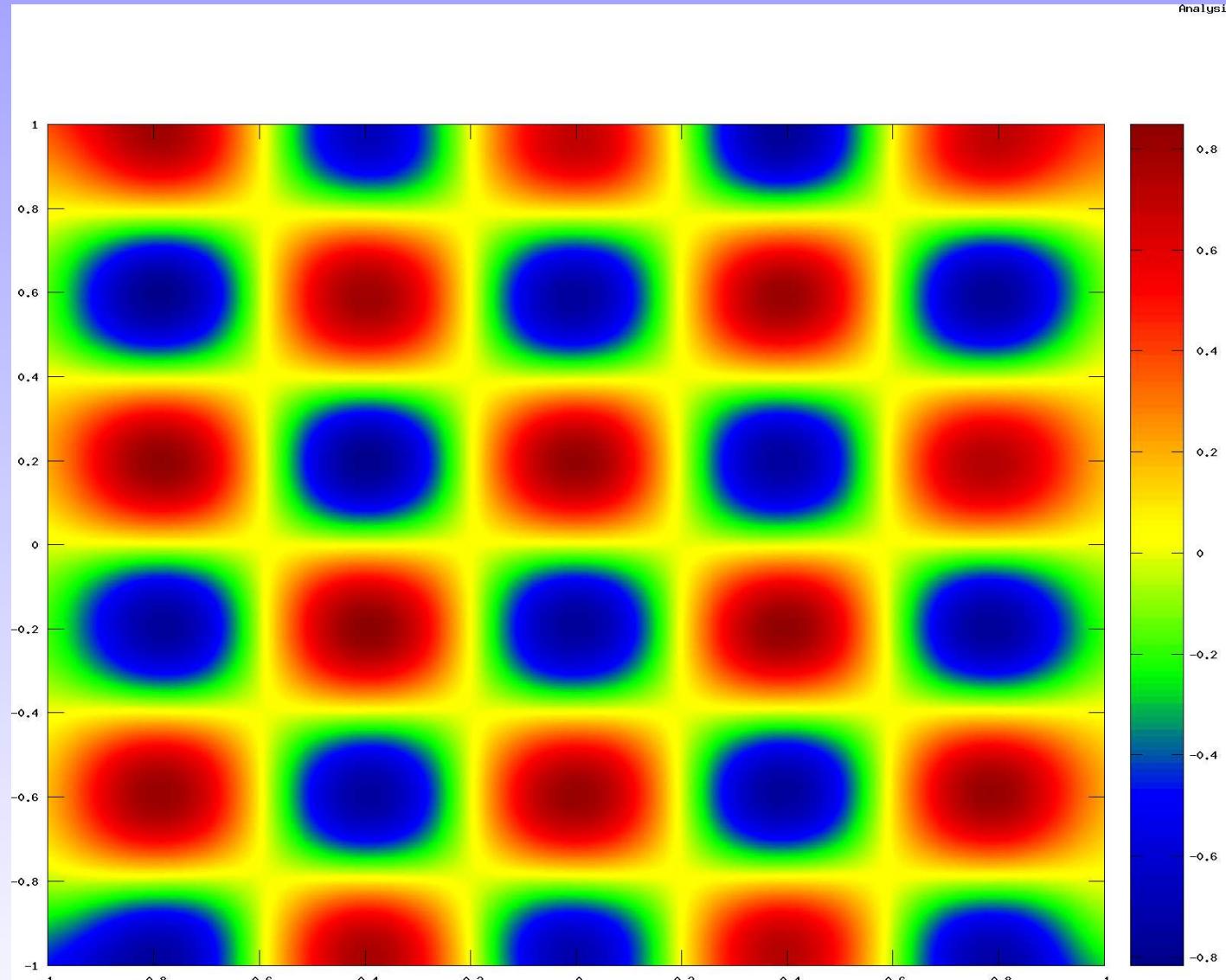
First analysis show a biais for each year's data with respect to first analysis. Subtract the biais estimate and redo the analysis, accumulating the biais. After convergence, detrended analysis+biais of the year.

Example without detrending



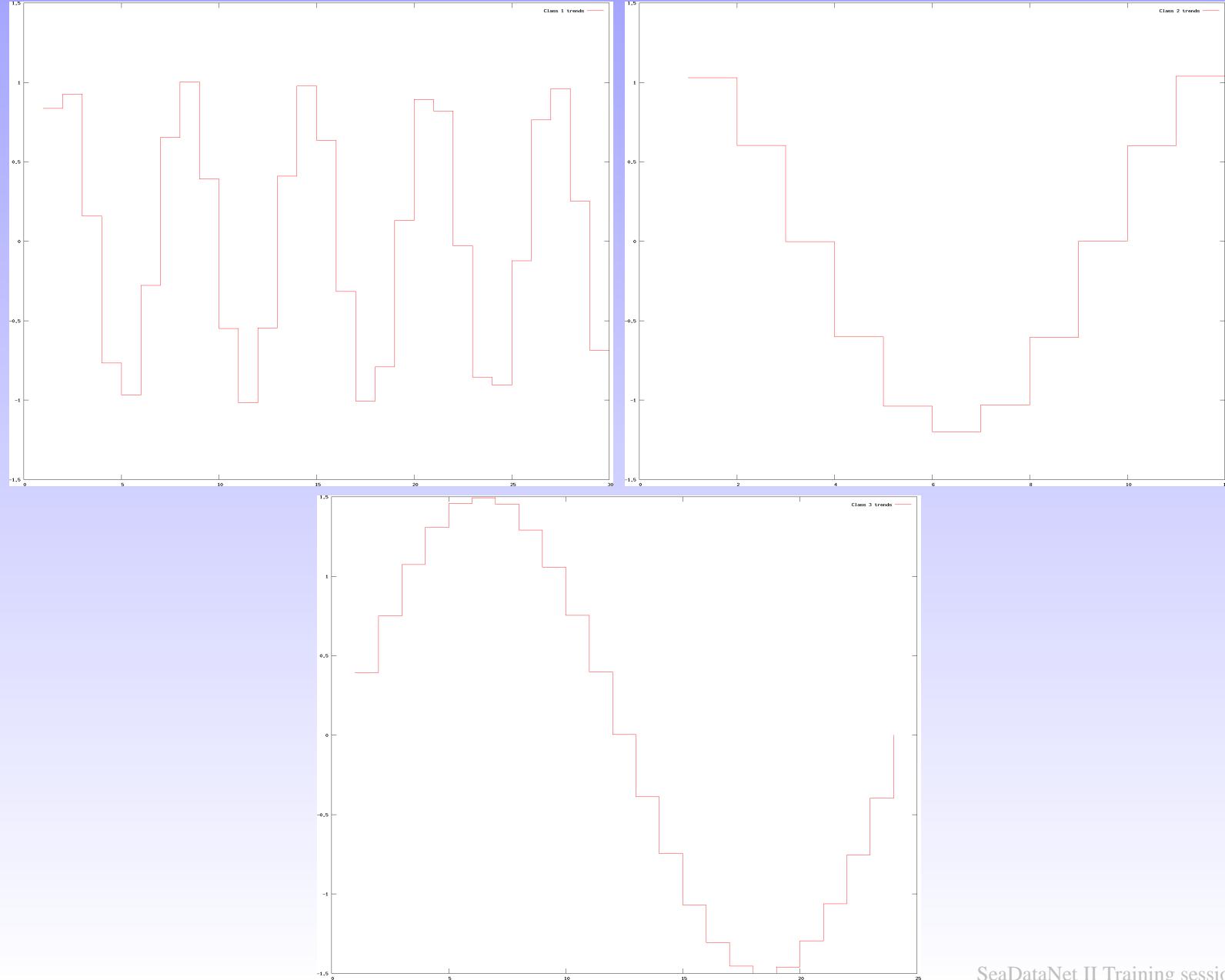
DIVA analysis of sin-cosine spatial structure with superimposed
decadal, seasonal and daily cycles and noise.

Example with detrending

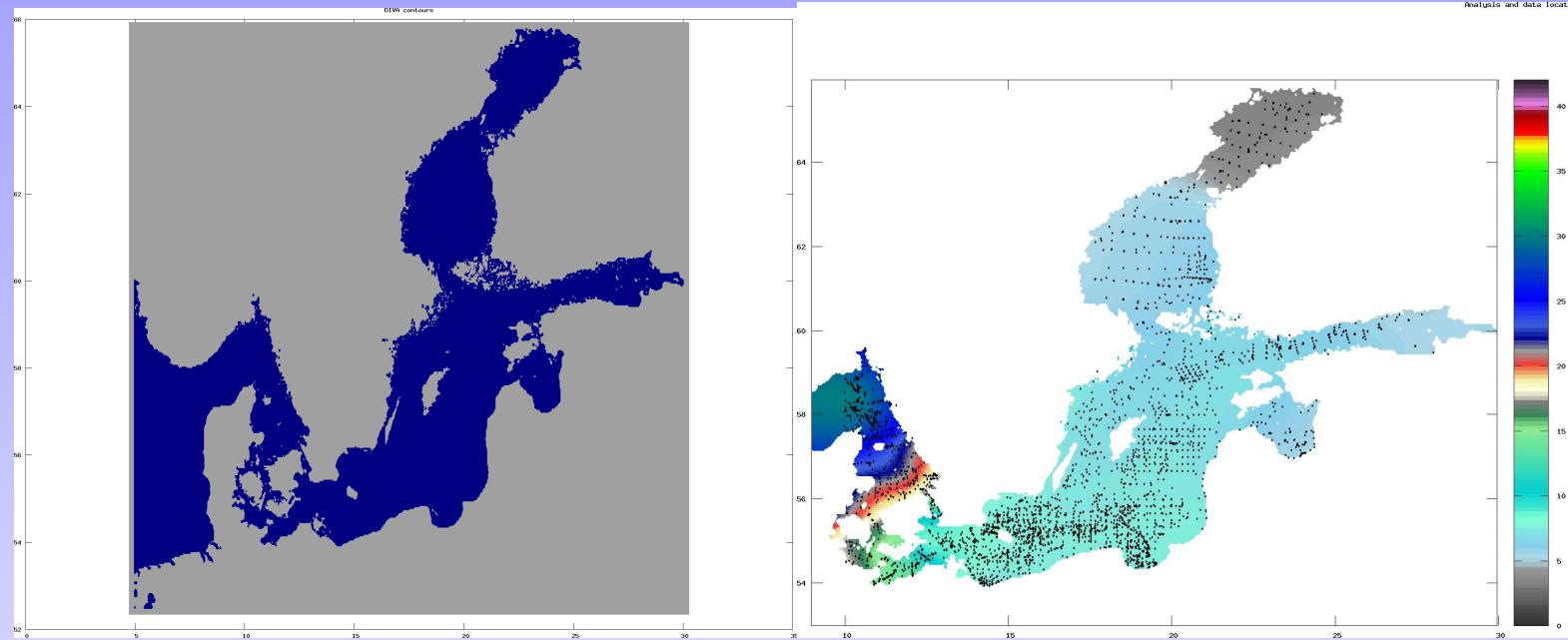


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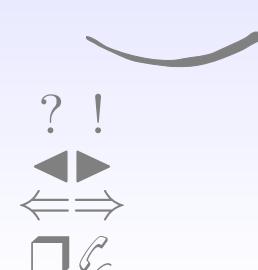
Trends can also be retrieved



Heterogeneous case

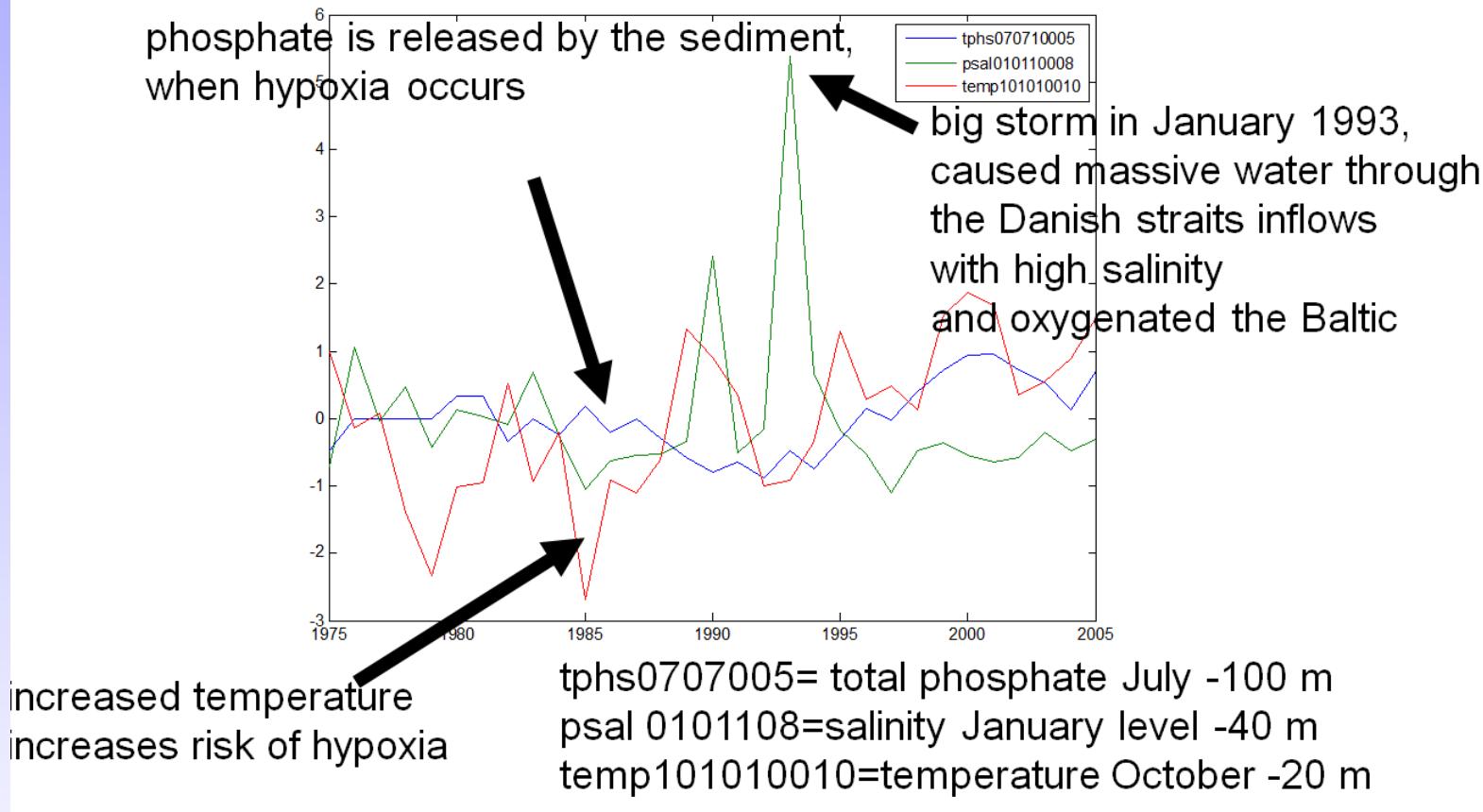


Baltic Salinity Climatology (Bassompierre *et al.*, 2010)



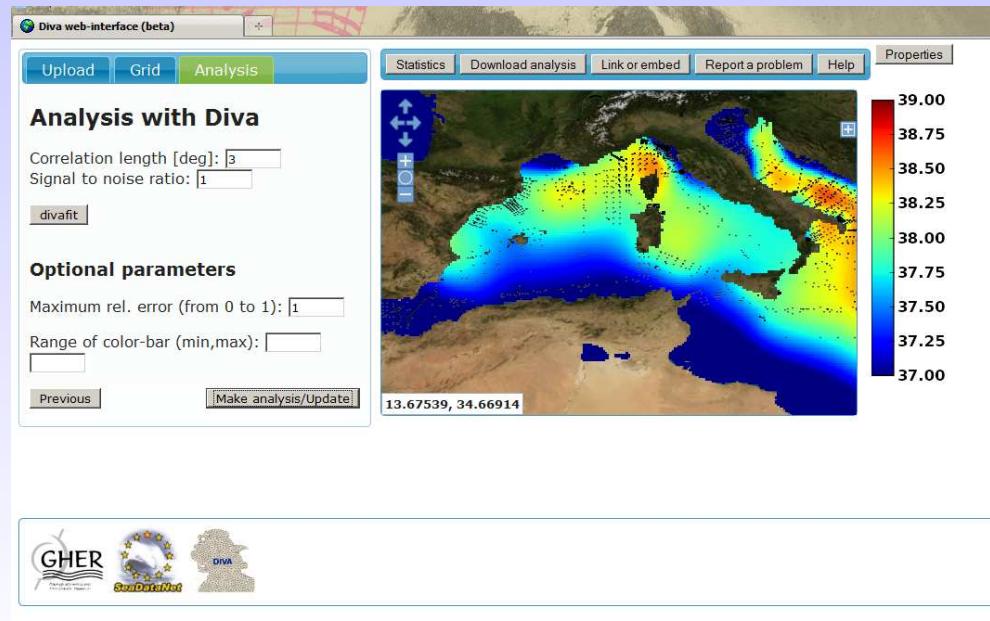
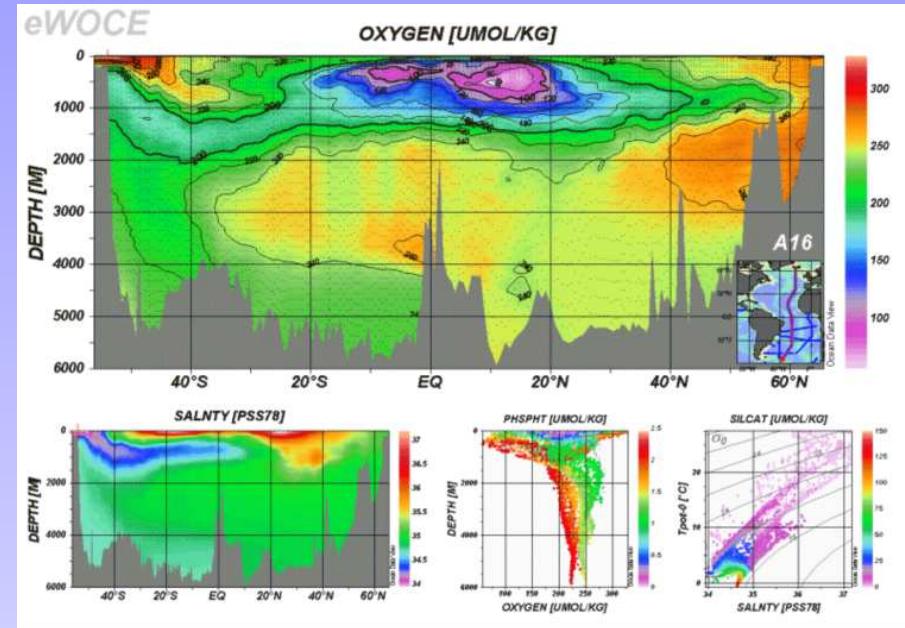
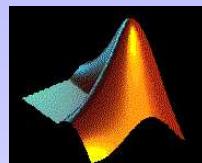
Heterogeneous case, trends

X= nutrient & climatic trends from Kattegat to Bothnian Bay (SDN products)



(Bassompierre *et al.*, 2010)

How to use DIVA?



```
~/cygdrive/c/jmb/cd-roms/diva-4.2.1/divastripped
CALL TO STORES MODULE: IPR = 1
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

Total nb. of pts where gridded solution is asked = 10201
Finished storing

$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
CALL TO GCUFAC MODULE: IPR = 1
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$

Trace average estimate: 0.0336772668
rms of misfits 1.32904057
MAXIMUM NUMBER OF INTEGER USED: 192895
MAXIMUM NUMBER OF REAL USED: 6775828
PRIOR ESTIMATE OF INTEGER USED: 888236
PRIOR ESTIMATE OF REAL USED: 9522239

oooooooooooooooooooooooooooooooooooooooooooooooooooo
D I U A - 4.2.2 - Execution Completed !
oooooooooooooooooooooooooooooooooooooooooooooooooooo
Output results for user

'fort.84' -> '../output/fieldgher.anl'
'fort.82' -> '../output/valatxascii.anl'
'fort.83' -> '../output/fieldascii.anl'
'fort.87' -> '../output/errorfieldgher.anl'
'fort.86' -> '../output/errorfieldascii.anl'
'fort.71' -> '../output/fieldatdpoint.anl'
'fort.77' -> '../output/gcuvall.dat'

Creation of file GridInfo.dat

'fort.87' -> '../output/ghertonetcdf/fort.87'
Creating netcdf file only for Field
since Varbal and ispec are 1 0
*** SUCCESS writing NetCDF file results.nc

-----
Analysis is finished
-----
```

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- *Implementations (14:00-15:00)*
- ***Diva-on-web (15:30-16:45)***
- *User feedback (16:45-17:00)*

Diva-on-web

<http://gher-diva.phys.ulg.ac.be/web-vis/diva.html>

Upload Grid Analysis

Upload observation

Text file ODV4

File: Parcourir...

Column separator: space or tab

Decimal separator: dot (.)

Format

The file must be an ASCII text file with three columns. The columns represent longitude, latitude and value of the observation respectively. For example:

```
29.7667 45.15 16.146
29.7667 45.15 16.346
...
```

[Sample global temperature data from ARGO](#)

[Next](#)

Statistics Download analysis Link or embed Report a problem Help





Diva-on-web

Data upload (3 column ascii file or ODV4) and output grid definition

Upload Grid Analysis Statistics Download analysis Link or embed Report a problem Help

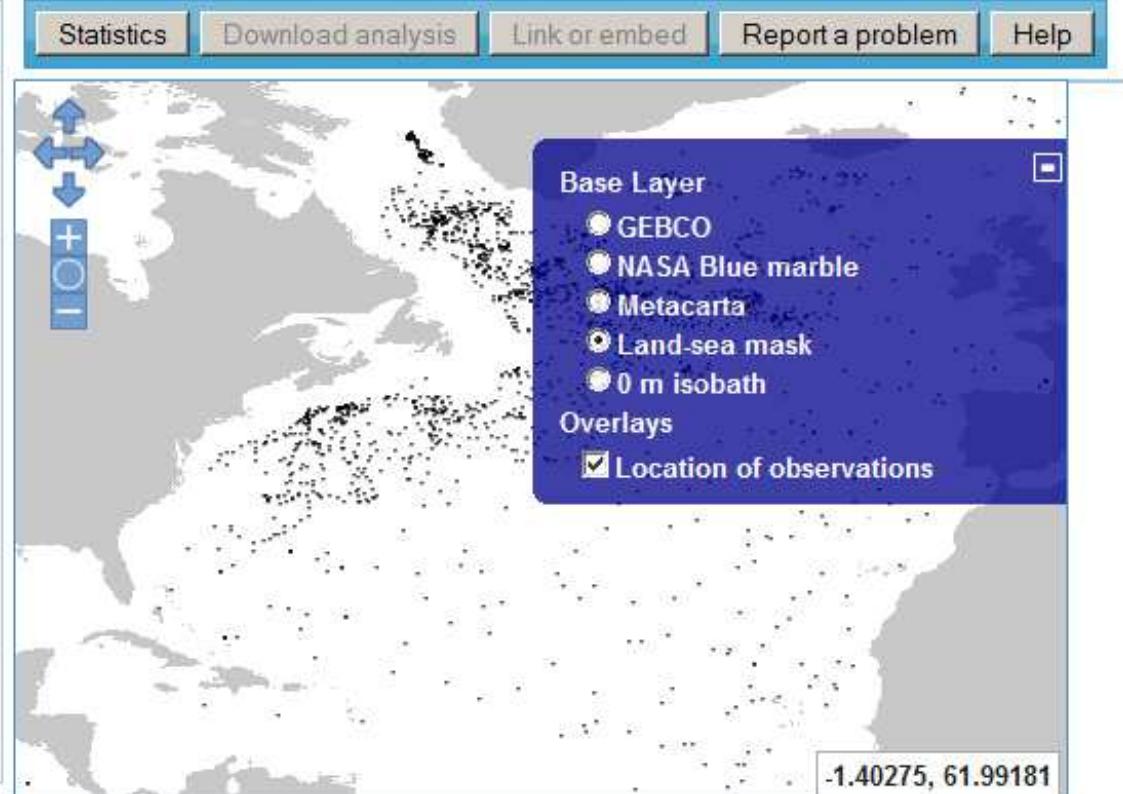
Grid coordinates

Longitude resolution: Latitude resolution:

Longitude range: Latitude range:

Depth level (m):

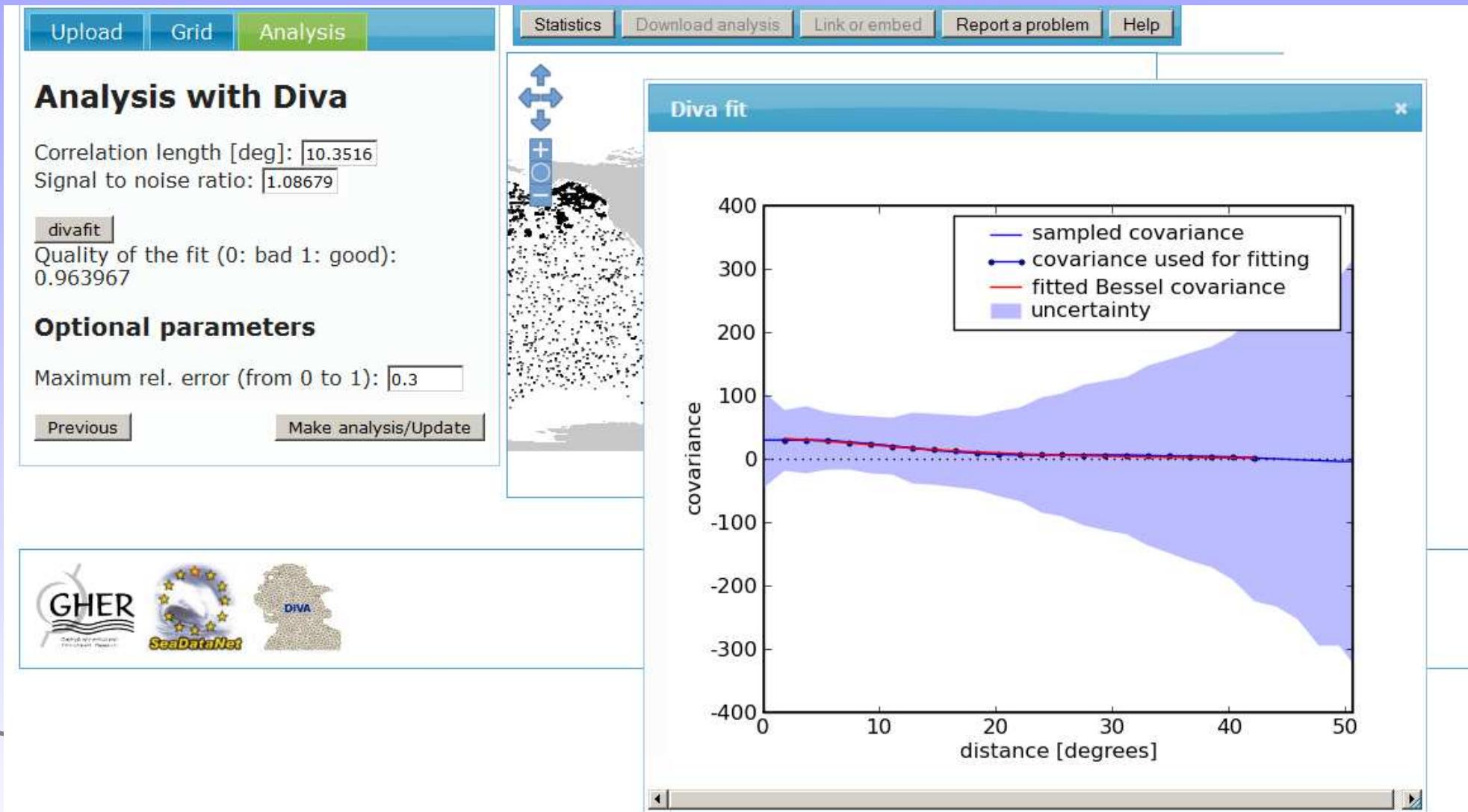
Bathymetric data base:





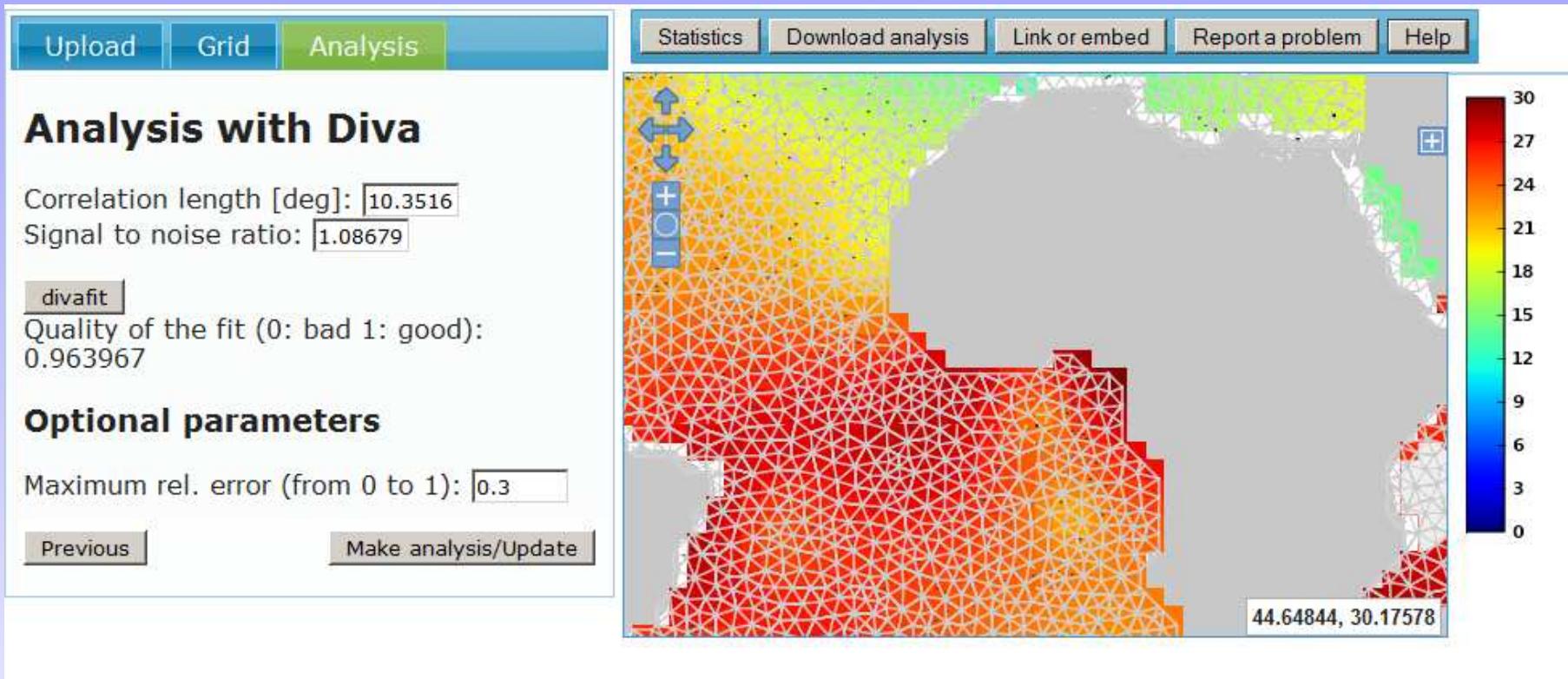
Diva-on-web

Analysis parameter definition (or fit)



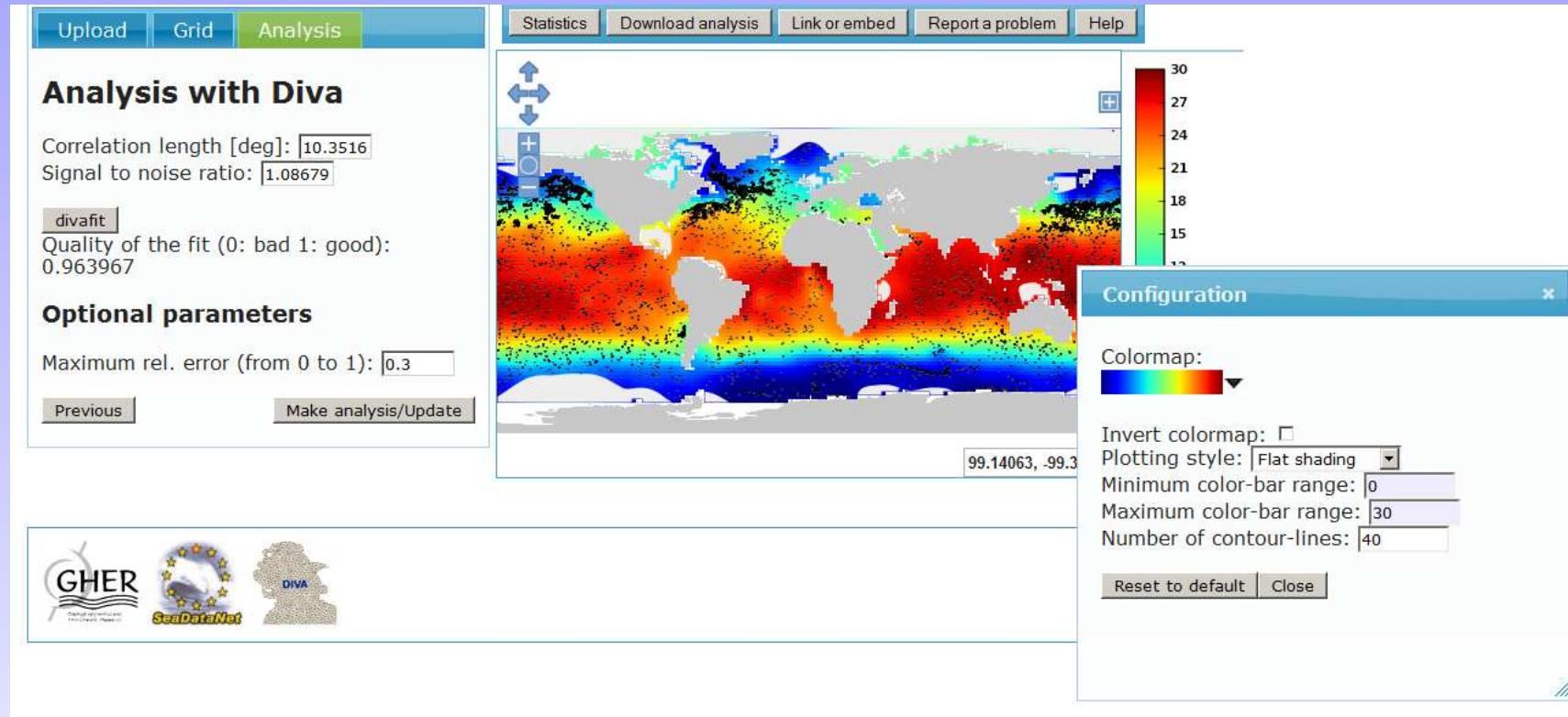
Diva-on-web

Analysis (and numerical grid)



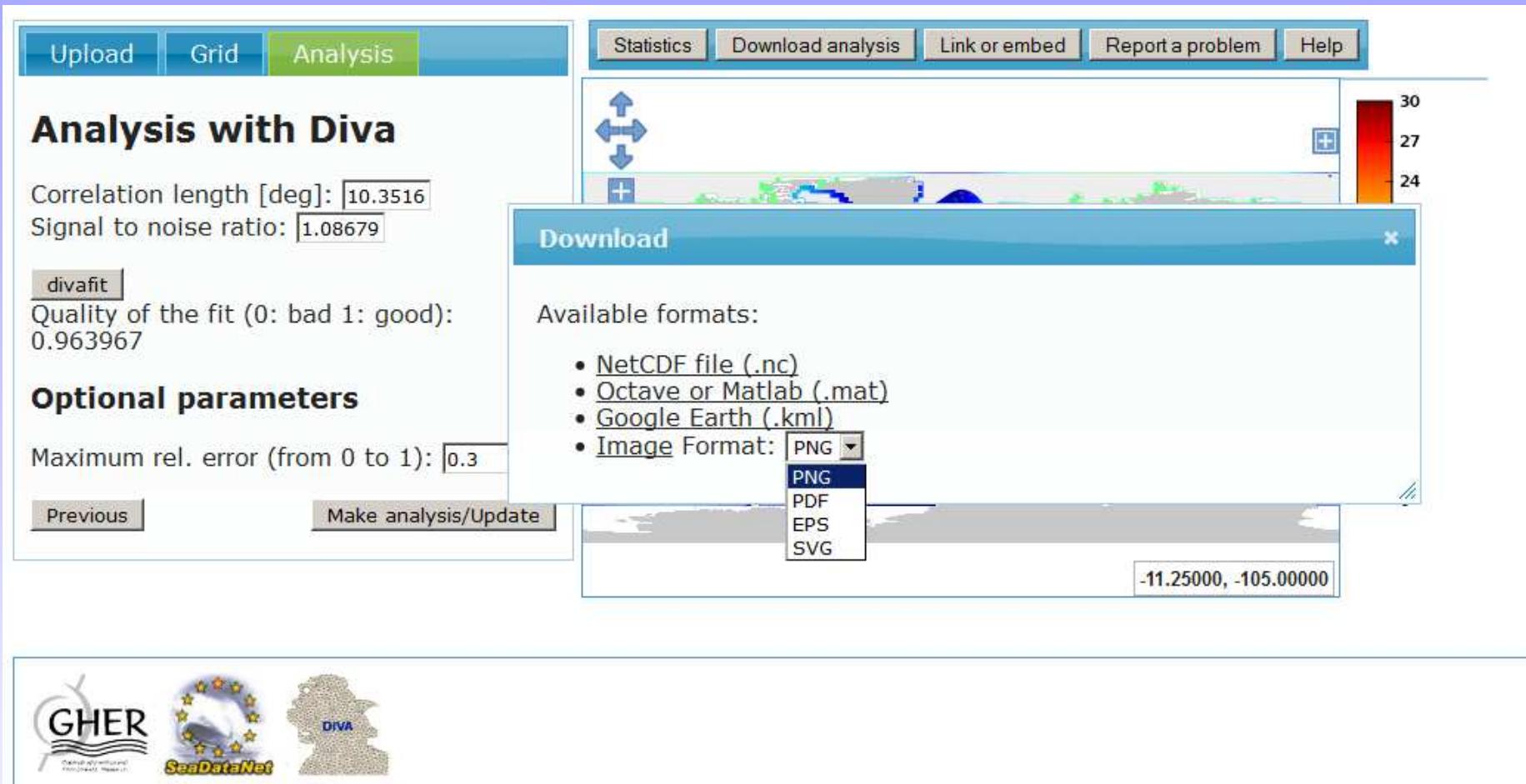
Diva-on-web

Plotting options



Diva-on-web

Download options



The screenshot shows the Diva-on-web interface with the "Analysis" tab selected. The main area displays analysis results for a dataset, including:

- Correlation length [deg]: 10.3516
- Signal to noise ratio: 1.08679
- divafit: Quality of the fit (0: bad 1: good): 0.963967
- Optional parameters: Maximum rel. error (from 0 to 1): 0.3

Below these are "Previous" and "Make analysis/Update" buttons.

A "Download" dialog box is open, listing available formats:

- NetCDF file (.nc)
- Octave or Matlab (.mat)
- Google Earth (.kml)
- Image Format: (dropdown menu showing options: PNG, PDF, EPS, SVG, currently set to PNG)

The bottom of the interface features logos for GHER, SeaDataNet, and DIVA.

Diva-on-web

Exploitation of WMS-OGC layering techniques

Upload Grid Analysis Statistics Download analysis Link or embed Report a problem Help

Analysis with Diva

Correlation length [deg]: 10.3516
Signal to noise ratio: 1.08679

divafit
Quality of the fit (0: bad 1: good): 0.963967

Optional parameters

Maximum rel. error (from 0 to 1)

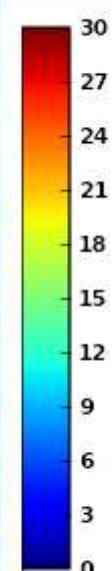
Previous Make ana

Link: <http://gher-diva.phys.ulg.be/>
Embed: <iframe src = "http://gher-

Copy and paste this line of code in your web page. To keep your results on our web-server, you must click on save below. We would appreciate if you leave your name and email address [\[more\]](#).

Name:
Email:

Save



Help

More information

http://modb.oce.ulg.ac.be/mediawiki/index.php/Using_Diva_on_web

The screenshot shows a web browser displaying a Mediawiki page titled "Using Diva on web". The page content is as follows:

Using Diva on web

Contents [hide]

- 1 What is the Diva web-interface
- 2 Using the Interface
 - 2.1 Upload
 - 2.1.1 Extracted data
 - 2.1.2 ODV4 files
 - 2.2 Grid
- 3 Analysis
- 4 Embed in your web site
- 5 Tested browsers
- 6 CPU time limit
- 7 Memory usage

What is the Diva web-interface

The Diva web-interface (available at <http://gher-diva.phys.ulg.ac.be/web-vis/diva.html>) is a web service to interpolate ocean data on a regular grid. It uses Data-Interpolating Variational Analysis (Diva) for the interpolation.

If you use the results generated by Diva-on-web, please include the following citation:

A. Barth, A. Alvera-Azcárate, C. Troupin, M. Ouberdous, and J.-M. Beckers. A web interface for gridding arbitrarily distributed *in situ* data based on Data-Interpolating Variational Analysis (Diva). *Advances in Geosciences*, 28:29–37, 2010. doi: 10.5194/adgeo-28-29-2010. URL <http://www.adv-geosci.net/28/29/2010/>.

Using the Interface

Upload

Extracted data

The file containing the *in situ* data must be an ASCII text file with three columns. The columns represent longitude, latitude and value of the observation respectively. For example:

```
29.7667 45.15 16.146
29.7667 45.15 16.346
29.7667 45.15 16.526
29.8167 45.15 2.016
...
```

The *in situ* data is thus extracted for a given time and depth (using programs such Ocean Data View).

ODV4 files

ODV4 files conforming to this standard (<http://www.seadatanet.org/Standards-Software/Data-Transport-Formats>) can also be used. In addition to the variable to analyse, the ODV4 file must have the columns named Station, Longitude, Latitude, Depth and time_ISO8601 (for time in yyyy-mm-ddThh:mm:ss.sss).

Multiple ODV4 files can also be placed in a zip file. All files in zip file are interpreted as ODV4 files. However files ending with metadata.csv are ignored.

Grid

- *ODV-DIVA gridding (12:00-12:30)*
- *Theory (13:30-14:00)*
- *Implementations (14:00-15:00)*
- *Diva-on-web (15:30-16:45)*
- ***User feedback (16:45-17:00)***

Feedback

- Report non-standard ODV4 files retrieved from SDN interface to SDN-TTT
- Report diva runtime errors (be it in ODV or WWW or stand-alone) to M.Ouberdous@ulg.ac.be
- For installation of GODIVA (4D batch mode), contact us
- When reporting problems in batch-mode version, try to pinpoint errors and minimal example which reproduces error (please do not come with "It does not work")
- For bugs on diva-on-web, use web interface for reporting
- Do not hesitate to add things to a wishlist
- Strong interactions during DIVA workshops

http://modb.oce.ulg.ac.be/mediawiki/index.php/Diva_workshop_2012





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